DIFFERENTIATING TRIGONOMETRIC FUNCTIONS OF ANGLES MEASURED IN DEGREES

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I found the short note *Inscribed polygon of maximal area* by Alexander Vaninsky (Mathematics Teacher, Vol. 99, No. 5, December 2005) very interesting as it presents a non–trivial application of the Lagrange multiplier rule to a problem with a significant historical value. (This was the method employed by Archimedes to approximate the area of a circle!) Even though the main result in the article is correct, I would like to point out an important correction in Vaninsky's derivation. The problem has to do with the use of degree measure for angles and the derivative of the sin function. The standard result from calculus that

$$\frac{\mathrm{d}(\sin x)}{\mathrm{d}x} = \cos x,$$

holds when the variable x measures angles in radians. Another standard result is the limit:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1,$$

where again, x is in radians.

If x is an angle measured in degrees, then the usual definition for the sine function of the angle x is:

$$\sin_d x = \sin\left(\frac{\pi}{180}\, x\right),$$

where the sine function on the right has domain the real numbers or radians. (We use the subscript "d" on the sine function of the left side to emphasize the difference between the sine function of angles in degrees and that for angles in radians.) It follows now using the chain rule that,

$$\frac{\mathrm{d}(\sin_d x)}{\mathrm{d}x} = \frac{\pi}{180}\,\cos\left(\frac{\pi}{180}\, x\right) = \frac{\pi}{180}\,\cos_d x.$$

Thus the last formula in Vaninsky's note should read

$$\cos_d \alpha_i = -\frac{360\lambda}{\pi R^2}.$$

If one changes in that article the equation for the restriction to:

$$\sum_{i=1}^{n} \alpha_i = 2\pi,$$

and the α's are interpreted as angles measured in radians, then all of the formulas in the article are correct. By the way, the limit

$$\lim_{x\to 0} \frac{\sin_d x}{x} = \frac{\pi}{180},$$

a fact that can easily be proved from the definition above for $\sin_d$ and the special limit mentioned at the beginning of this note. This result can also be conjectured by computing the ratio `(sin x)/x` for `x` small, with a standard calculator on *degree mode*.