# November 2000
# Course 1

Society of Actuaries/Casualty Actuarial Society

**1.** A recent study indicates that the annual cost of maintaining and repairing a car in a town in Ontario averages 200 with a variance of 260 .

If a tax of 20% is introduced on all items associated with the maintenance and repair of cars (*i.e.*, everything is made 20% more expensive), what will be the variance of the annual cost of maintaining and repairing a car?

(A) 208

(B) 260

(C) 270

(D) 312

(E) 374

**2.** An investor purchases two assets, each having an initial value of 1000 . The value $V_n$ of the first asset after $n$ years can be modeled by the relationship:

$$V_n = 1.10 V_{n-1} \quad \text{for } n = 1, 2, 3, \ldots$$

The value $W_n$ of the second asset after $n$ years can be modeled by the relationship:

$$W_n = W_{n-1} + 0.20 W_0 \quad \text{for } n = 1, 2, 3, \ldots$$

According to these models, by how much will the value of the first asset exceed the value of the second asset after 25 years?

(A) 4050

(B) 4835

(C) 5035

(D) 5718

(E) 6000

**3.** An auto insurance company has 10,000 policyholders. Each policyholder is classified as

(i) young or old;

(ii) male or female; and

(iii) married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

How many of the company's policyholders are young, female, and single?

(A) 280

(B) 423

(C) 486

(D) 880

(E) 896

**4.** A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let $X$ denote the disease state of a patient, and let $Y$ denote the outcome of the diagnostic test. The joint probability function of $X$ and $Y$ is given by:

$$P(X = 0, Y = 0) = 0.800$$

$$P(X = 1, Y = 0) = 0.050$$

$$P(X = 0, Y = 1) = 0.025$$

$$P(X = 1, Y = 1) = 0.125$$

Calculate $\operatorname{Var}(Y \mid X = 1)$ .

(A) 0.13

(B) 0.15

(C) 0.20

(D) 0.51

(E) 0.71

**5.** An equation of the line tangent to the graph of a differentiable function $f$ at $x = 0$ is $y = 3x + 4$ .

Determine $\lim_{x \to 0} \frac{x\, f(x)}{\sin(2x)}$ .

(A) 0

(B) 1

(C) 2

(D) 4

(E) The limit does not exist.

**6.** An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2 . Assume the numbers of claims filed by distinct policyholders are independent of one another.

What is the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

(A) 0.68

(B) 0.82

(C) 0.87

(D) 0.95

(E) 1.00

**7.** A group insurance policy covers the medical claims of the employees of a small company. The value, $V$, of the claims made in one year is described by

$$V = 100{,}000Y$$

where $Y$ is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4 & \text{for } 0 < y < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

What is the conditional probability that $V$ exceeds 40,000, given that $V$ exceeds 10,000 ?

(A) 0.08

(B) 0.13

(C) 0.17

(D) 0.20

(E) 0.51

**8.** An insurance company can sell 20 auto insurance policies per month if it charges 40 per policy. Moreover, for each decrease or increase of 1 in the price per policy, the company can sell 1 more or 1 less policy, respectively. Fixed costs are 100 . Variable costs are 32 per policy.

What is the maximum monthly profit that the insurance company can achieve from selling auto insurance policies?

(A) 96

(B) 196

(C) 296

(D) 400

(E) 900

**9.** An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100 . Losses incurred follow an exponential distribution with mean 300 .

What is the $95^{\text{th}}$ percentile of actual losses that exceed the deductible?

(A) 600

(B) 700

(C) 800

(D) 900

(E) 1000

**10.** Let $S$ be a solid in 3-space and $f$ a function defined on $S$ such that:

$$\iiint_S f(x,y,z)\,dV = 5$$

$$\iiint_S (4f(x,y,z)+3)\,dV = 47$$

Calculate the volume of $S$ .

(A) 2

(B) 5

(C) 7

(D) 9

(E) 14

**11.** An actuary determines that the claim size for a certain class of accidents is a random variable, $X$, with moment generating function

$$M_X(t) = \frac{1}{(1-2500t)^4} .$$

Determine the standard deviation of the claim size for this class of accidents.

(A) 1,340

(B) 5,000

(C) 8,660

(D) 10,000

(E) 11,180

**12.** An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

| Type of driver | Percentage of all drivers | Probability of at least one collision |
|---|---|---|
| Teen | 8% | 0.15 |
| Young adult | 16% | 0.08 |
| Midlife | 45% | 0.04 |
| Senior | 31% | 0.05 |
| Total | 100% | |

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

(A) 0.06

(B) 0.16

(C) 0.19

(D) 0.22

(E) 0.25

**13.** An actuary believes that the demand for life insurance, $L$, and the demand for health insurance, $H$, can be modeled as functions of time, $t$:

$$L(t) = t^3 + 9t + 100 \quad \text{for } 0 \le t \le 4$$

$$H(t) = 6t^2 + 102 \qquad \text{for } 0 \le t \le 4$$

During the time period $0 \le t \le 4$, the greatest absolute difference between the two demands occurs $n$ times.

Determine $n$ .

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

**14.** A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount $x$ if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made.

At what level must $x$ be set if the expected payment made under this insurance is to be 1000 ?

(A) 3858

(B) 4449

(C) 5382

(D) 5644

(E) 7235

**15.** Let $C$ be the curve in $\mathbf{R}^3$ defined by $x = t^2$, $y = 4t^{3/2}$, $z = 9t$, for $t \geq 0$ .

Calculate the distance along $C$ from (1, 4, 9) to (16, 32, 36) .

(A) 6

(B) 33

(C) 42

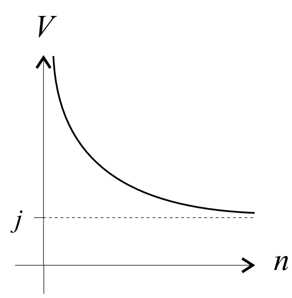
(D) 52

(E) 597

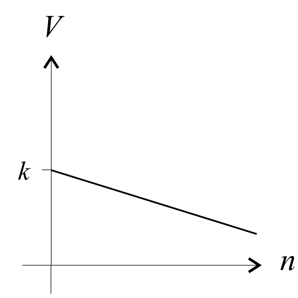
**16.** The total cost of manufacturing $n$ microchips consists of a positive fixed set-up cost of $k$ plus a constant positive cost $j$ per microchip manufactured.

Which of the following most closely represents the graph of $V$, the average cost per microchip?

(A)




(B)

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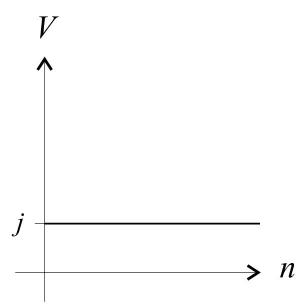


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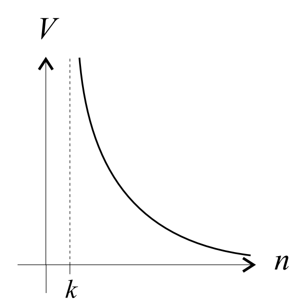
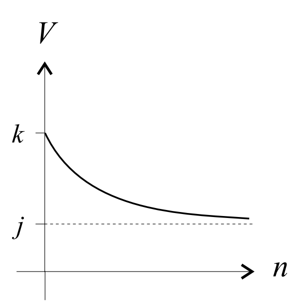
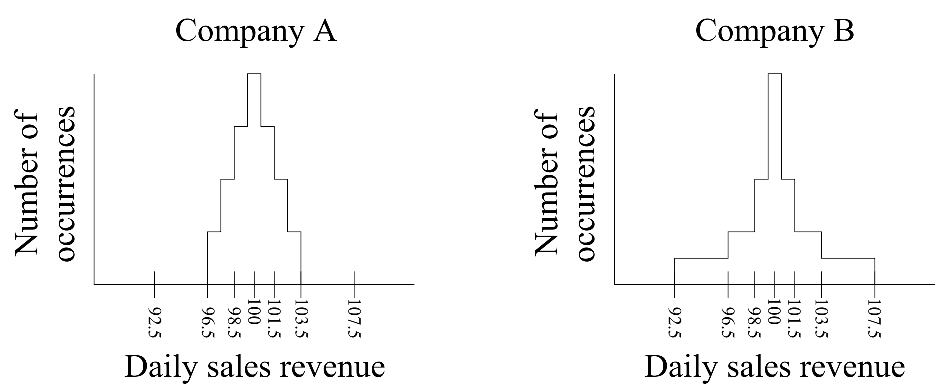



(E)



**17.** A stock market analyst has recorded the daily sales revenue for two companies over the last year and displayed them in the histograms below.




The analyst noticed that a daily sales revenue above 100 for Company A was always accompanied by a daily sales revenue below 100 for Company B, and vice versa.

Let $X$ denote the daily sales revenue for Company A and let $Y$ denote the daily sales revenue for Company B, on some future day.

Assuming that for each company the daily sales revenues are independent and identically distributed, which of the following is true?

(A) $\operatorname{Var}(X) > \operatorname{Var}(Y)$ and $\operatorname{Var}(X+Y) > \operatorname{Var}(X) + \operatorname{Var}(Y)$ .

(B) $\operatorname{Var}(X) > \operatorname{Var}(Y)$ and $\operatorname{Var}(X+Y) < \operatorname{Var}(X) + \operatorname{Var}(Y)$ .

(C) $\operatorname{Var}(X) > \operatorname{Var}(Y)$ and $\operatorname{Var}(X+Y) = \operatorname{Var}(X) + \operatorname{Var}(Y)$ .

(D) $\operatorname{Var}(X) < \operatorname{Var}(Y)$ and $\operatorname{Var}(X+Y) > \operatorname{Var}(X) + \operatorname{Var}(Y)$ .

(E) $\operatorname{Var}(X) < \operatorname{Var}(Y)$ and $\operatorname{Var}(X+Y) < \operatorname{Var}(X) + \operatorname{Var}(Y)$ .

**18.** Due to decreasing business, the amount an insurer expects to pay for claims will decrease at a constant rate of 5% per month indefinitely. This month the insurer paid 1000 in claims.

What is the insurer's total expected amount of claims to be paid over the 30-month period that began this month?

(A) 13,922

(B) 14,707

(C) 14,922

(D) 15,707

(E) 15,922

**19.** Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000 .

What is the probability that the average of 25 randomly selected claims exceeds 20,000 ?

(A) 0.01

(B) 0.15

(C) 0.27

(D) 0.33

(E) 0.45

**20.** The future lifetimes (in months) of two components of a machine have the following joint density function:

$$f(x,y)=\begin{cases}\dfrac{6}{125,000}(50-x-y) & \text{for } 0<x<50-y<50\\ 0 & \text{otherwise.}\end{cases}$$

What is the probability that both components are still functioning 20 months from now?

(A) $\dfrac{6}{125,000}\displaystyle\int_0^{20}\int_0^{20}(50-x-y)\,dy\,dx$

(B) $\dfrac{6}{125,000}\displaystyle\int_{20}^{30}\int_{20}^{50-x}(50-x-y)\,dy\,dx$

(C) $\dfrac{6}{125,000}\displaystyle\int_{20}^{30}\int_{20}^{50-x-y}(50-x-y)\,dy\,dx$

(D) $\dfrac{6}{125,000}\displaystyle\int_{20}^{50}\int_{20}^{50-x}(50-x-y)\,dy\,dx$

(E) $\dfrac{6}{125,000}\displaystyle\int_{20}^{50}\int_{20}^{50-x-y}(50-x-y)\,dy\,dx$

**21.** A consumer has 100 to spend on $x$ units of product X and $y$ units of product Y . The price per unit is 10 for X and 5 for Y .

The consumer prefers quantities (including fractional quantities) $x_1$ and $y_1$ over $x_2$ and $y_2$ if

$$f(x_1,y_1) > f(x_2, y_2), \quad \text{where}$$
$$f(x,y) = x^{0.75}y^{0.25} \qquad \text{for } x, y \geq 0 .$$

What is the maximum value of $f(x,y)$ that can be achieved given the consumer's spending constraint?

(A) 6.78

(B) 7.50

(C) 8.41

(D) 9.58

(E) 11.40

**22.** The probability that a randomly chosen male has a circulation problem is 0.25 . Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem.

What is the conditional probability that a male has a circulation problem, given that he is a smoker?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{1}{2}$

(E) $\frac{2}{3}$

**23.** A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5 .

What is the expected amount paid to the company under this policy during a one-year period?

(A) 2,769

(B) 5,000

(C) 7,231

(D) 8,347

(E) 10,578

**24.** Let $f$ be a function such that $f(x+h) - f(x) = 6xh + 3h^2$ and $f(1) = 5$ .

Determine $f(2) - f'(2)$ .

(A) 0

(B) 2

(C) 3

(D) 5

(E) 6

**25.** A manufacturer's annual losses follow a distribution with density function

$$f(x) = \begin{cases} \dfrac{2.5(0.6)^{2.5}}{x^{3.5}} & \text{for } x > 0.6 \\ 0 & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2 .

What is the mean of the manufacturer's annual losses not paid by the insurance policy?

(A) 0.84

(B) 0.88

(C) 0.93

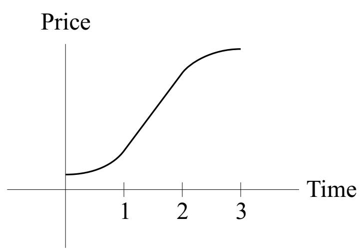
(D) 0.95

(E) 1.00

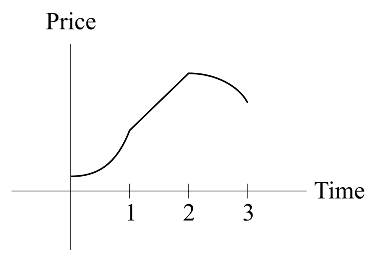
**26.** The price of gasoline changes over time. Over a period of three years, the rate of change in price increases for the first year, remains constant for the second year, and declines for the third year. The rate of change in price is never negative over this time.

Which of the following graphs best represents price graphed against time?

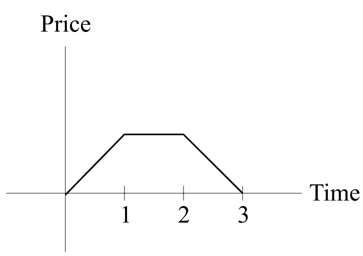
(A)



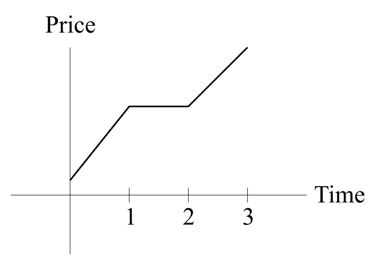

(B)



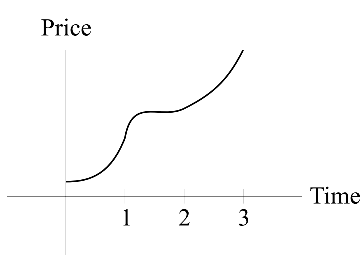

(C)




(D)




(E)



**27.** Let $X_1, X_2, X_3$ be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} \frac{1}{3} & \text{for } x = 0 \\ \frac{2}{3} & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the moment generating function, M($t$), of $Y = X_1X_2X_3$ .

(A) $\frac{19}{27} + \frac{8}{27}e^t$

(B) $1 + 2e^t$

(C) $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$

(D) $\frac{1}{27} + \frac{8}{27}e^{3t}$

(E) $\frac{1}{3} + \frac{2}{3}e^{3t}$

**28.** A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.

(ii) 22% have low blood pressure.

(iii) 15% have an irregular heartbeat.

(iv) Of those with an irregular heartbeat, one-third have high blood pressure.

(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure?

(A) 2%

(B) 5%

(C) 8%

(D) 9%

(E) 20%

**29.** Insurance losses are not always reported in the year they occur. In fact, some losses are reported many years later. The year in which a loss occurs is called the occurrence year.

For a given occurrence year, let $R_n$ denote the total number of losses reported in the occurrence year and the following $n$ years. An actuary determines that $R_n$ can be modeled by the sequence:

$$R_{n+1} = 2^{0.75^n} R_n \quad \text{for } n = 0, 1, 2, \ldots$$

For occurrence year 1999, 250 losses were reported during 1999. In other words, $R_0 = 250$ .

How many more occurrence year 1999 losses does the model predict will be reported in years subsequent to 1999?

(A) 1750

(B) 2000

(C) 3172

(D) 3422

(E) 3750

**30.** An actuary studying the insurance preferences of automobile owners makes the following conclusions:

(i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.

(ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

(iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15 .

What is the probability that an automobile owner purchases neither collision nor disability coverage?

(A) 0.18

(B) 0.33

(C) 0.48

(D) 0.67

(E) 0.82

**31.** Let $f(x) = \begin{cases} 3x^2 & \text{for } 0 \le x \le 1 \\ 4 - x & \text{for } 1 \le x \le 4 \end{cases}.$

Let $R$ be the region bounded by the graph of $f$, the $x$-axis, and the lines $x = b$ and $x = b + 2$, where $0 \le b \le 1$.

Determine the value of $b$ that maximizes the area of $R$.

(A) $0$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

(E) $1$

**32.** The monthly profit of Company I can be modeled by a continuous random variable with density function $f$. Company II has a monthly profit that is twice that of Company I.

Determine the probability density function of the monthly profit of Company II.

(A) $\frac{1}{2} f\left(\frac{x}{2}\right)$

(B) $f\left(\frac{x}{2}\right)$

(C) $2 f\left(\frac{x}{2}\right)$

(D) $2 f(x)$

(E) $2 f(2x)$

**33.** Let $C$ be the curve defined by the polar function $r = 2 + \cos(\theta)$ . The vertices of triangle $PQR$ are the points on $C$ corresponding to $\theta = 0$, $\theta = \pi$, and $\theta = \dfrac{\pi}{3}$.

Calculate the area of triangle $PQR$ .

(A) $2$

(B) $\dfrac{5\sqrt{3}}{4}$

(C) $\dfrac{5}{2}$

(D) $4$

(E) $\dfrac{5\sqrt{3}}{2}$

**34.** The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.

What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

(A) 0.15

(B) 0.34

(C) 0.43

(D) 0.57

(E) 0.66

**35.** A company's value at time $t$ is growing at a rate proportional to the difference between 20 and its value at $t$ .

At $t = 0$, the value is 2 . At $t = 1$, the value is 3 .

Calculate the value at $t = 3$ .

(A) 4.84

(B) 5.00

(C) 5.87

(D) 6.39

(E) 6.75

**36.** An insurance company insures a large number of drivers. Let $X$ be the random variable representing the company's losses under collision insurance, and let $Y$ represent the company's losses under liability insurance. $X$ and $Y$ have joint density function

$$f(x,y)=\begin{cases}\dfrac{2x+2-y}{4} & \text{for } 0<x<1 \text{ and } 0<y<2\\ 0 & \text{otherwise.}\end{cases}$$

What is the probability that the total loss is at least 1 ?

(A) 0.33

(B) 0.38

(C) 0.41

(D) 0.71

(E) 0.75

**37.** The level of prices, $P$, is determined by the level of employment, $E$, and the cost of raw materials, $M$, as follows:

$$P = 160\, E^{3/4}\, M^{2/5}$$

Which of the following statements is true?

(A) $P$ increases at a constant rate as either $E$ or $M$ increases.

(B) $P$ increases at a decreasing rate as $E$ increases, but increases at an increasing rate as $M$ increases.

(C) $P$ increases at an increasing rate as $E$ increases, but increases at a decreasing rate as $M$ increases.

(D) $P$ increases at an increasing rate as either $E$ or $M$ increases.

(E) $P$ increases at a decreasing rate as either $E$ or $M$ increases.

**38.** The profit for a new product is given by $Z = 3X - Y - 5$. $X$ and $Y$ are independent random variables with $\mathrm{Var}(X) = 1$ and $\mathrm{Var}(Y) = 2$.

What is the variance of $Z$?

(A) 1

(B) 5

(C) 7

(D) 11

(E) 16

**39.** In a certain town, the number of deaths in year $t$ due to a particular disease is modeled by

$\frac{90,000}{(t+3)^3}$ for $t = 1, 2, 3 \ldots$

Let $N$ be the total number of deaths that the model predicts will occur in the town after the end of the 27th year.

Which of the following intervals contains $N$ ?

(A) $39.5 \le N < 43.0$

(B) $43.0 \le N < 46.5$

(C) $46.5 \le N < 50.0$

(D) $50.0 \le N < 53.5$

(E) $53.5 \le N < 57.0$

**40.** A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails.

Let $X$ and $Y$ be the times at which the first and second circuits fail, respectively. $X$ and $Y$ have joint probability density function

$$f(x,y)=\begin{cases}6e^{-x}e^{-2y} & \text{for } 0<x<y<\infty\\ 0 & \text{otherwise.}\end{cases}$$

What is the expected time at which the device fails?

(A) 0.33

(B) 0.50

(C) 0.67

(D) 0.83

(E) 1.50

# Course 1
# November 2000
# Answer Key

| | | | |
|---|---|---|---|
| 1 | E | 21 | A |
| 2 | B | 22 | C |
| 3 | D | 23 | C |
| 4 | C | 24 | B |
| 5 | C | 25 | C |
| 6 | B | 26 | A |
| 7 | B | 27 | A |
| 8 | A | 28 | E |
| 9 | E | 29 | E |
| 10 | D | 30 | B |
| 11 | B | 31 | C |
| 12 | D | 32 | A |
| 13 | D | 33 | E |
| 14 | D | 34 | C |
| 15 | C | 35 | A |
| 16 | A | 36 | D |
| 17 | E | 37 | E |
| 18 | D | 38 | D |
| 19 | C | 39 | C |
| 20 | B | 40 | D |

1. E

Let X and Y denote the annual cost of maintaining and repairing a car before and after the 20% tax, respectively. Then Y = 1.2X and Var[Y] = Var[1.2X] = $(1.2)^2$ Var[X] = $(1.2)^2(260) = 374$ .

2. B

Note that $V_{25} = 1.10\ V_{24} = (1.10)^2\ V_{23} = \ldots = (1.10)^{25}\ V_0 = (1.10)^{25}\ 1000$

$= 10{,}835$ while $W_{25} = W_{24} + 0.20\ W_0 = W_{23} + (2)(0.20)\ W_0 = \ldots = W_0 + (25)(0.20)W_0$

$= 1000 + 25(0.20)(1000) = 6000$ . Therefore, $V_{25} - W_{25} = 10{,}835 - 6{,}000 = 4835$ .

3. D

Let N(C) denote the number of policyholders in classification C . Then

N(Young $\cap$ Female $\cap$ Single) = N(Young $\cap$ Female) – N(Young $\cap$ Female $\cap$ Married)
= N(Young) – N(Young $\cap$ Male) – [N(Young $\cap$ Married) – N(Young $\cap$ Married $\cap$ Male)] = 3000 – 1320 – (1400 – 600) = 880 .

4. C

Note that

$$P(Y = 0 \mid X = 1) = \frac{P(X=1, Y=0)}{P(X=1)} = \frac{P(X=1,Y=0)}{P(X=1,Y=0)+P(X=1,Y=1)} = \frac{0.05}{0.05+0.125}$$

$= 0.286$

$P(Y = 1 \mid X=1) = 1 - P(Y = 0 \mid X = 1) = 1 - 0.286 = 0.714$

Therefore, $E(Y \mid X = 1) = (0)\ P(Y = 0 \mid X = 1) + (1)\ P(Y = 1 \mid X = 1) = (1)(0.714) = 0.714$

$E(Y^2 \mid X = 1) = (0)^2\ P(Y = 0 \mid X = 1) + (1)^2\ P(Y = 1 \mid X = 1) = 0.714$

$Var(Y \mid X = 1) = E(Y^2 \mid X = 1) - [E(Y \mid X = 1)]^2 = 0.714 - (0.714)^2 = 0.20$

5. C

Note that f(0) = 3(0) + 4 = 4 since y = 3x + 4 is tangent to f at x = 0 .

Moreover, $\lim_{x\to 0} f(x) = f(0) = 4$ since f is differentiable at x = 0 .

Finally, from the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, it follows that

$$\lim_{x\to 0}\frac{xf(x)}{\sin(2x)} = \lim_{x\to 0}\frac{2xf(x)}{2\sin(2x)} = \frac{1}{2}\lim_{x\to 0} f(x) \Big/ \lim_{x\to 0}\frac{\sin(2x)}{2x} = \frac{4}{2} = 2 .$$

6. B

Let $X_1, \dots, X_{1250}$ be the number of claims filed by each of the 1250 policyholders. We are given that each $X_i$ follows a Poisson distribution with mean 2 . It follows that $E[X_i] = Var[X_i] = 2$ . Now we are interested in the random variable $S = X_1 + \dots + X_{1250}$ . Assuming that the random variables are independent, we may conclude that S has an approximate normal distribution with E[S] = Var[S] = (2)(1250) = 2500 .

Therefore P[2450 < S < 2600] =

$$P\left[\frac{2450-2500}{\sqrt{2500}} < \frac{S-2500}{\sqrt{2500}} < \frac{2600-2500}{\sqrt{2500}}\right] = P\left[-1 < \frac{S-2500}{50} < 2\right]$$

$$= P\left[\frac{S-2500}{50} < 2\right] - P\left[\frac{S-2500}{50} < -1\right]$$

Then using the normal approximation with $Z = \frac{S-2500}{50}$, we have P[2450 < S < 2600]

≈ P[Z < 2] – P[Z > 1] = P[Z < 2] + P[Z < 1] – 1 ≈ 0.9773 + 0.8413 – 1 = 0.8186 .

7. B

To determine k, note that

$$1 = \int_0^1 k(1-y)^4\,dy = -\frac{k}{5}(1-y)^5 \Big|_0^1 = \frac{k}{5}$$

k = 5

We next need to find P[V > 10,000] = P[100,000 Y > 10,000] = P[Y > 0.1]

$$= \int_{0.1}^1 5(1-y)^4 dy = -(1-y)^5 \Big|_{0.1}^1 = (0.9)^5 = 0.59 \text{ and } P[V > 40,000]$$

$$= P[100,000\ Y > 40,000] = P[Y > 0.4] = \int_{0.4}^1 5(1-y)^4 dy = -(1-y)^5 \Big|_{0.4}^1 = (0.6)^5 = 0.078\ .$$

It now follows that P[V > 40,000 | V > 10,000]

$$= \frac{P[V > 40,000 \cap V > 10,000]}{P[V > 10,000]} = \frac{P[V > 40,000]}{P[V > 10,000]} = \frac{0.078}{0.590} = 0.132\ .$$

8. A

Let x be the number of policies sold per month. Then the price function p(x) satisfies the relationship x = 20 + [40 – p(x)] . Therefore, p(x) = 60 – x . Next, define R(x), C(x), and P(x) to be the company's respective revenue, cost, and profit functions. Then

$R(x) = x\,p(x) = x(60 - x) = 60x - x^2$

C(x) = 32x + 100

$P(x) = R(x) - C(x) = 60x - x^2 - 32x - 100 = -x^2 + 28x - 100$ . Now since the quadratic P(x) is clearly maximized at x such that P′(x) = 0, we see

0 = P′(x) = –2x + 28

2x = 28 or x = 14 .

Finally, $P(14) = -(14)^2 + 28(14) - 100 = 96$ is the maximum profit the company can achieve.

9. E

Let X denote actual losses incurred. We are given that X follows an exponential distribution with mean 300, and we are asked to find the 95th percentile of all claims that exceed 100 . Consequently, we want to find $p_{95}$ such that

$0.95 = \dfrac{\Pr[100 < x < p_{95}]}{P[X > 100]} = \dfrac{F(p_{95}) - F(100)}{1 - F(100)}$ where F(x) is the distribution function of X .

Now $F(x) = 1 - e^{-x/300}$ .

Therefore, $0.95 = \dfrac{1 - e^{-p_{95}/300} - (1 - e^{-100/300})}{1 - (1 - e^{-100/300})} = \dfrac{e^{-1/3} - e^{-p_{95}/300}}{e^{-1/3}} = 1 - e^{1/3} e^{-p_{95}/300}$

$e^{-p_{95}/300} = 0.05\, e^{-1/3}$

$p_{95} = -300 \ln(0.05\, e^{-1/3}) = 999$

10. D

We are given that:

$\iiint\limits_S f(x,y,z)dV = 5$ and $\iiint\limits_S [4f(x,y,z) + 3]dV = 47$

Using these two equalities, we can solve for the volume of S, $\iiint\limits_S dV$, as follows:

$47 = \iiint\limits_S [4f(x,y,z) + 3]dV = 4\iiint\limits_S f(x,y,z)dV + 3\iiint\limits_S dV$

$47 = 4(5) + 3\iiint\limits_S dV$

$\iiint\limits_S dV = (1/3)(47 - 20) = 9$ .

11. B

We are given that $M_x(t) = \dfrac{1}{(1 - 2500t)^4}$ for the claim size X in a certain class of accidents.

First, compute $M_x'(t) = \dfrac{(-4)(-2500)}{(1-2500t)^5} = \dfrac{10,000}{(1-2500t)^5}$

$M_x''(t) = \dfrac{(10,000)(-5)(-2500)}{(1-2500t)^6} = \dfrac{125,000,000}{(1-2500t)^6}$

Then $E[X] = M_x'(0) = 10,000$

$E[X^2] = M_x''(0) = 125,000,000$

$Var[X] = E[X^2] - \{E[X]\}^2 = 125,000,000 - (10,000)^2 = 25,000,000$

$\sqrt{Var[X]} = 5,000$ .

12. D

Let

C = Event of a collision

T = Event of a teen driver

Y = Event of a young adult driver

M = Event of a midlife driver

S = Event of a senior driver

Then using Bayes' Theorem, we see that

$$P[Y \mid C] = \frac{P[C|Y]P[Y]}{P[C|T]P[T]+P[C|Y]P[Y]+P[C|M]P[M]+P[C|S]P[S]}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08)+(0.08)(0.16)+(0.04)(0.45)+(0.05)(0.31)} = 0.22 .$$

13. D

In order to determine n, we need to determine the extreme values of

$D(t) = L(t) - H(t) = t^3 + 9t + 100 - 6t^2 - 102 = t^3 - 6t^2 + 9t - 2,\ 0 \le t \le 4$ .

Note: $D'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t - 1)(t - 3)$ and recall that extreme values of D(t) occur either at t such that $D'(t) = 0$ or at endpoints of the interval $0 \le t \le 4$ on which D(t) is defined. Extreme values can thus occur at t = 0, 1, 3, or 4 . The table below provides the remaining information we need in order to determine n:

| t | D(t) | $\lvert D(t) \rvert$ |
|---|---|---|
| 0 | –2 | 2 |
| 1 | 2 | 2 |
| 3 | –2 | 2 |
| 4 | 2 | 2 |

We conclude that n = 4 .

14. D

Let T be the time from purchase until failure of the equipment. We are given that T is exponentially distributed with parameter $\lambda = 10$ since $10 = E[T] = \lambda$ . Next define the payment

P under the insurance contract by $P = \begin{cases} x & \text{for } 0 \le T \le 1 \\ \dfrac{x}{2} & \text{for } 1 < T \le 3 \\ 0 & \text{for } T > 3 \end{cases}$

We want to find x such that

$$1000 = E[P] = \int_0^1 \frac{x}{10} e^{-t/10}\, dt + \int_1^3 \frac{x}{2}\frac{1}{10} e^{-t/10}\, dt = -xe^{-t/10}\Big|_0^1 - \frac{x}{2}e^{-t/10}\Big|_1^3$$

$= -x e^{-1/10} + x - (x/2) e^{-3/10} + (x/2) e^{-1/10} = x(1 - ½ e^{-1/10} - ½ e^{-3/10}) = 0.1772x$ .

We conclude that x = 5644 .

15. C

First, observe that the point (1,4,9) corresponds to t = 1 while the point (16,32,36) corresponds to t = 4 . Next, let L denote the distance along C from (1,4,9) to (16,32,36) . Then:

$$L = \int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt = \int_1^4 \sqrt{(2t)^2 + \left(6t^{1/2}\right)^2 + (9)^2}\,dt = \int_1^4 \sqrt{4t^2 + 36t + 81}\,dt$$

$$= \int_1^4 \sqrt{(2t+9)^2}\,dt = \int_1^4 (2t+9)\,dt = \left(t^2 + 9t\right)\Big|_1^4 = 16 + 36 - (1 + 9) = 42 .$$

16. A

Let C(n) denote the total cost of manufacturing n widgets, and let V(n) denote the average cost of manufacturing n widgets.

We are given that C(n) = jn + k, from which it follows that V(n) = (1/n) C(n) = j + k/n . Next note that $\lim_{n\to\infty}$ V(n) = j and $\lim_{n\to 0}$ V(n) = +∞ . The graph that most closely reflects these characteristics of V is A .

17. E

The histograms imply that Var(X) < Var(Y) because Company A's share price X is less dispersed about the mean share price of 100 than Company B's share price Y . Moreover, we can deduce that Cov(X,Y) < 0 from the fact that a share price above 100 for Company A is always accompanied by a share price less than 100 for Company B . Since Var(X + Y) = Var(X) + Var(Y) + 2 Cov(X,Y) we conclude that Var(X + Y) < Var(X) + Var(Y) .

18. D

Let C be expected claims. Then C = 1000 + 1000 (0.95) + 1000 $(0.95)^2$ + . . . + 1000 $(0.95)^{29}$ = 1000 [1 + 0.95 + $(0.95)^2$ + . . . + $(0.95)^{29}$] = 1000 $\dfrac{1-(0.95)^{30}}{1-0.95}$ = 15,707 .

19. C

Let $X_1, \ldots, X_{25}$ denote the 25 collision claims, and let $\bar{X} = \frac{1}{25}(X_1 + \ldots + X_{25})$ . We are given that each $X_i$ (i = 1, . . . , 25) follows a normal distribution with mean 19,400 and standard deviation 5000 . As a result $\bar{X}$ also follows a normal distribution with mean 19,400 and standard deviation $\frac{1}{\sqrt{25}}$ (5000) = 1000 . We conclude that P[ $\bar{X}$ > 20,000]

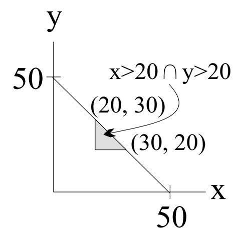
$$= P\left[\frac{\bar{X} - 19{,}400}{1000} > \frac{20{,}000 - 19{,}400}{1000}\right] = P\left[\frac{\bar{X} - 19{,}400}{1000} > 0.6\right] = 1 - \Phi(0.6) = 1 - 0.7257$$

= 0.2743 .

20. B

We are given that $f(x,y) = \begin{cases} \dfrac{6}{125{,}000}(50 - x - y) & \text{for } 0 < x < 50 - y < 50 \\ 0 & \text{otherwise} \end{cases}$

and we want to determine P[X > 20 ∩ Y > 20] . In order to determine integration limits, consider the following diagram:




We conclude that P[X > 20 ∩ Y > 20] = $\dfrac{6}{125{,}000}\displaystyle\int_{20}^{30}\int_{20}^{50-x}(50 - x - y)\, dy\, dx$ .

21. A

The consumer's spending constraints are characterized by the equation 10x + 5y = 100 or y = 20 – 2x . The constrained maximum value of f(x,y) can thus be found by focusing on $g(x) = f(x, 20 - 2x) = x^{0.75}(20 - 2x)^{0.25}$, $0 \le x \le 10$ . Given this restriction of x to the closed interval [0,10], a maximum for g(x) clearly exists and occurs at the interval's endpoints or at x such that $0 = g'(x) = 0.75\, x^{-0.25}(20 - 2x)^{0.25} - 2(0.25)\, x^{0.75}(20 - 2x)^{-0.75}$ $= 0.25\, x^{-0.25}(20 - 2x)^{-0.75}[3(20 - 2x) - 2x] = 0.25\, x^{-0.25}(20 - 2x)^{-0.75}(60 - 8x)$ . This condition is satisfied when 0 = 60 – 8x or x = 7.5 . Therefore, the constrained maximum of f(x,y) occurs at x = 0, 7.5, or 10 . Since g(0) = g(10) = 0, we conclude that $g(7.5) = (7.5)^{0.75}[20 - 2(7.5)]^{0.25} = (7.5)^{0.75}(5)^{0.25} = 6.78$ is the maximum value of f(x,y) subject to the consumer's spending constraints.

22. C

Let:
S = Event of a smoker
C = Event of a circulation problem
Then we are given that P[C] = 0.25 and $P[S|C] = 2\,P[S|C^C]$

Now applying Bayes' Theorem, we find that $P[C|S] = \dfrac{P[S|C]P[C]}{P[S|C]P[C] + P[S|C^C](P[C^C])}$

$$= \frac{2P[S|C^C]P[C]}{2P[S|C^C]P[C] + P[S|C^C](1 - P[C])} = \frac{2(0.25)}{2(0.25) + 0.75} = \frac{2}{2+3} = \frac{2}{5} .$$

23. C

Let N be the number of major snowstorms per year, and let P be the amount paid to the company under the policy. Then Pr[N = n] = $\frac{(3/2)^n e^{-3/2}}{n!}$, n = 0, 1, 2, . . . and

$$P = \begin{cases} 0 & \text{for } N = 0 \\ 10,000(N-1) & \text{for } N \geq 1 \end{cases}.$$

Now observe that E[P] = $\sum_{n=1}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}$

= 10,000 $e^{-3/2}$ + $\sum_{n=0}^{\infty} 10,000(n-1)\frac{(3/2)^n e^{-3/2}}{n!}$ = 10,000 $e^{-3/2}$ + E[10,000 (N – 1)]

= 10,000 $e^{-3/2}$ + E[10,000N] – E[10,000] = 10,000 $e^{-3/2}$ + 10,000 (3/2) – 10,000 = 7,231 .

24. B

First rewrite the equation as $\frac{f(x+h)-f(x)}{h}$ = 6x + 3h . Then, taking the limit as h → 0, we see that f ′(x) = $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ = 6x . It follows that f(x) = ∫ 6x dx = $3x^2$ + C and, since f(1) = 5, we have 5 = $3(1)^2$ + C or C = 2 . Therefore, f(x) = $3x^2$ + 2 and f(2) – f ′(2) = $3(2)^2$ + 2 – 6(2) = 2 .

25. C

Let Y denote the manufacturer's retained annual losses.

Then $Y = \begin{cases} x & \text{for } 0.6 < x \leq 2 \\ 2 & \text{for } x > 2 \end{cases}$

and E[Y] = $\int_{0.6}^{2} x\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx + \int_{2}^{\infty} 2\left[\frac{2.5(0.6)^{2.5}}{x^{3.5}}\right]dx = \int_{0.6}^{2}\frac{2.5(0.6)^{2.5}}{x^{2.5}}dx - \frac{2(0.6)^{2.5}}{x^{2.5}}\Big|_{2}^{\infty}$

$$= -\frac{2.5(0.6)^{2.5}}{1.5x^{1.5}}\Big|_{0.6}^{2} + \frac{2(0.6)^{2.5}}{(2)^{2.5}} = -\frac{2.5(0.6)^{2.5}}{1.5(2)^{1.5}} + \frac{2.5(0.6)^{2.5}}{1.5(0.6)^{1.5}} + \frac{(0.6)^{2.5}}{2^{1.5}} = 0.9343 .$$

26. A

Let p denote price, and let t denote time. We are given:

i) $d^2p/dt^2 > 0$ during the first year (i.e., the price curve is concave up for 0 < t < 1);
ii) $d^2p/dt^2 = 0$ during the second year (i.e., the price curve is linear for 1 < t < 2);
iii) $d^2p/dt^2 < 0$ during the third year (i.e., the price curve is concave down for 2 < t < 3); and
iv) $dp/dt \geq 0$ during all three years (i.e., the price curve is nondecreasing for 0 ≤ t ≤ 3) .

Only graph A is consistent with the above requirements.

27. A

Let g(y) be the probability function for $Y = X_1X_2X_3$. Note that Y = 1 if and only if $X_1 = X_2 = X_3 = 1$. Otherwise, Y = 0. Since $P[Y = 1] = P[X_1 = 1 \cap X_2 = 1 \cap X_3 = 1]$ $= P[X_1 = 1]\,P[X_2 = 1]\,P[X_3 = 1] = (2/3)^3 = 8/27$.

We conclude that $g(y) = \begin{cases} \dfrac{19}{27} & \text{for } y = 0 \\ \dfrac{8}{27} & \text{for } y = 1 \\ 0 & \text{otherwise} \end{cases}$

and M(t) = $E\left[e^{y_t}\right] = \dfrac{19}{27} + \dfrac{8}{27}e^t$

28. E

"Boxed" numbers in the table below were computed.

| | High BP | Low BP | Norm BP | Total |
|---|---|---|---|---|
| Regular heartbeat | 0.09 | 0.20 | 0.56 | 0.85 |
| Irregular heartbeat | 0.05 | 0.02 | 0.08 | 0.15 |
| Total | 0.14 | 0.22 | 0.64 | 1.00 |

From the table, we can see that 20% of patients have a regular heartbeat and low blood pressure.

29. E

We first need to determine $\lim_{n\to\infty} R_n$. To do this, observe that

$R_0 = 250$

$R_1 = 2\,R_0 = 2(250)$

$R_2 = 2^{0.75}\,R_1 = (2)(2^{0.75})(250)$

$R_3 = 2^{0.75^2}\,R_2 = (2)(2^{0.75})(2^{0.75})^2(250) = 2^{1+0.75+(0.75)^2}\,(250)$

$\vdots$

$R_n = 2^{1+0.75+(0.75)^2+\ldots+(0.75)^{n-1}}\,(250) = 2^{\frac{1-(0.75)^n}{1-0.75}}\,(250)$

Therefore, $\lim_{n\to\infty} R_n = 2^{1/(1-0.75)}\,(250) = 2^4\,(250) = 4000$

Finally, we are asked to find $\lim_{n\to\infty} R_n - R_0 = 4000 - 250 = 3750$.

30. B

Let

C = Event that a policyholder buys collision coverage

D = Event that a policyholder buys disability coverage

Then we are given that $P[C] = 2P[D]$ and $P[C \cap D] = 0.15$ .

By the independence of C and D, it therefore follows that

$0.15 = P[C \cap D] = P[C]\, P[D] = 2P[D]\, P[D] = 2(P[D])^2$

$(P[D])^2 = 0.15/2 = 0.075$

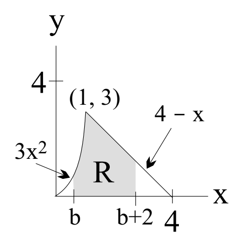
$P[D] = \sqrt{0.075}$ and $P[C] = 2P[D] = 2\sqrt{0.075}$

Now the independence of C and D also implies the independence of $C^C$ and $D^C$ . As a result, we see that $P[C^C \cap D^C] = P[C^C]\, P[D^C] = (1 - P[C])\,(1 - P[D])$

$= (1 - 2\sqrt{0.075})\,(1 - \sqrt{0.075}) = 0.33$ .

31. C

The graph of f(x) is shown below:




Note that for $0 \le b \le 1$,

$$\text{Area(R)} = \int_b^1 3x^2 dx + \int_1^{b+2} (4-x)dx = -\int_1^b 3x^2 dx + \int_1^{b+2} (4-x)dx$$

By the Fundamental Theorem of Calculus,

$d/db\,[\text{Area(R)}] = -3b^2 + [4 - (b + 2)]\, d/db\,[b + 2] = -3b^2 + 2 - b = -(3b^2 + b - 2)$

$= -(3b - 2)(b + 1)$ . Also, $d^2/db^2\,[\text{Area(R)}] = -6b - 1 < 0$ for $0 \le b \le 1$ . It follows that Area(R) is concave down as a function of b on the interval $0 \le b \le 1$ .

Since $d/db\,[\text{Area(R)}] = 0$ at $b = 2/3$, the concavity of Area(R) implies that $b = 2/3$ maximizes Area(R) on the interval $0 \le b \le 1$ .

32. A

Let X and Y be the monthly profits of Company I and Company II, respectively. We are given that the pdf of X is f . Let us also take g to be the pdf of Y and take F and G to be the distribution functions corresponding to f and g . Then $G(y) = Pr[Y \le y] = P[2X \le y]$ $= P[X \le y/2] = F(y/2)$ and $g(y) = G'(y) = d/dy\, F(y/2) = ½\, F'(y/2) = ½\, f(y/2)$ .

33. E

We will first find the vertex corresponding to each $\theta$ .

I. $\theta = 0$

$r = 2 + \cos(0) = 3$, $\cos(0) = 1$, $\sin(0) = 0$

vertex: $(r\cos\theta, r\sin\theta) = (3, 0)$

II. $\theta = \pi$

$r = 2 + \cos(\pi) = 1$, $\cos(\pi) = -1$, $\sin(\pi) = 0$

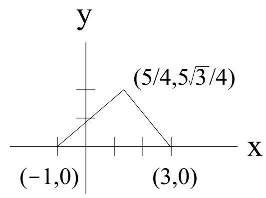
vertex: $(r\cos\theta, r\sin\theta) = (-1, 0)$

III. $\theta = \pi/3$

$r = 2 + \cos(\pi/3) = 5/2$, $\cos(\pi/3) = ½$, $\sin(\pi/3) = \sqrt{3}/2$

vertex: $(r\cos\theta, r\sin\theta) = (5/4, (5\sqrt{3}/4)$

The diagram below shows triangle PQR and thus summarizes the work above.




It follows from the diagram that

Area of Triangle = ½ (base)(height) = $½ (4)(5\sqrt{3}/4) = (5\sqrt{3}/2)$ .

34. C

Let T denote the number of days that elapse before a high-risk driver is involved in an accident. Then T is exponentially distributed with unknown parameter $\lambda$ . Now we are given that

$$0.3 = P[T \le 50] = \int_0^{50} \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_0^{50} = 1 - e^{-50\lambda}$$

Therefore, $e^{-50\lambda} = 0.7$ or $\lambda = -(1/50)\ln(0.7)$

It follows that $P[T \le 80] = \int_0^{80} \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_0^{80} = 1 - e^{-80\lambda}$

$= 1 - e^{(80/50)\ln(0.7)} = 1 - (0.7)^{80/50} = 0.435$ .

35. A

Let y(t) denote the company's value at time t .
We are given that $y'(t) = k[20 - y(t)]$, $y(0) = 2$, $y(1) = 3$, k is constant.
The solution to this differential equation may be found as follows:

$$\int \frac{dy(t)}{20 - y(t)} = \int k dt$$

$-\ln [20 - y(t)] = kt + C$ (C is a constant)
$20 - y(t) = e^{-kt - C}$
$y(t) = 20 - e^{-kt - C} = 20 - e^{-C} e^{-kt}$
Now using the fact that $y(0) = 2$, we see that
$2 = y(0) = 20 - e^{-C}$
$e^{-C} = 18$
It follows that $y(t) = 20 - 18 e^{-kt}$
Then using the fact that $y(1) = 3$, we see that
$3 = y(1) = 20 - 18 e^{-k}$
$18e^{-k} = 17$
$e^{-k} = 17/18$
$-k = \ln (17/18)$
It follows that $y(t) = 20 - 18 e^{t \ln(17/18)} = 20 - 18(17/18)^t$
We conclude that $y(3) = 20 - 18 (17/18)^3 = 4.84$ .

36. D

We want to find $P[X + Y > 1]$ . To this end, note that $P[X + Y > 1]$

$$= \int_0^1 \int_{1-x}^2 \left[\frac{2x+2-y}{4}\right] dy dx = \int_0^1 \left[\frac{1}{2}xy + \frac{1}{2}y - \frac{1}{8}y^2\right]_{1-x}^2 dx$$

$$= \int_0^1 \left[x + 1 - \frac{1}{2} - \frac{1}{2}x(1-x) - \frac{1}{2}(1-x) + \frac{1}{8}(1-x)^2\right] dx = \int_0^1 \left[x + \frac{1}{2}x^2 + \frac{1}{8} - \frac{1}{4}x + \frac{1}{8}x^2\right] dx$$

$$= \int_0^1 \left[\frac{5}{8}x^2 + \frac{3}{4}x + \frac{1}{8}\right] dx = \left[\frac{5}{24}x^3 + \frac{3}{8}x^2 + \frac{1}{8}x\right]_0^1 = \frac{5}{24} + \frac{3}{8} + \frac{1}{8} = \frac{17}{24}$$

37. E

The first and second partial derivatives of P with respect to E and M are given below:
$P = 160\, E^{3/4} M^{2/5}$, $E > 0$, $M > 0$
$\partial P/\partial E = 120\, E^{-1/4} M^{2/5} > 0$ $\qquad \partial P/\partial M = 64\, E^{3/4} M^{-3/5} > 0$
$\partial^2 P/\partial E^2 = -30\, E^{-5/4} M^{2/5} < 0$ $\qquad \partial^2 P/\partial M^2 = -38.4\, E^{3/4} M^{-8/5} < 0$
It follows that P increases at a decreasing rate as either E or M increases.

38. D

Note that due to the independence of X and Y
$Var(Z) = Var(3X - Y - 5) = Var(3X) + Var(Y) = 3^2\, Var(X) + Var(Y) = 9(1) + 2 = 11$ .

39. C

Consider the following two diagrams:

Diagram 1

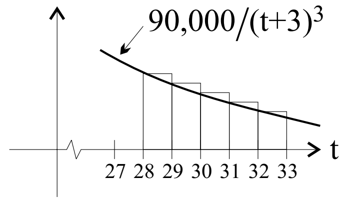
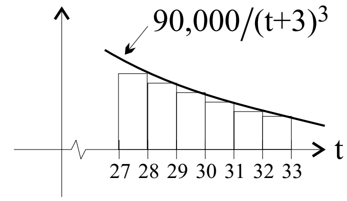



Diagram 2




Note that Diagram 1 shows that $\sum_{t=28}^{\infty} \frac{90,000}{(t+3)^3} > \int_{28}^{\infty} \frac{90,000}{(t+3)^3} dt = -\frac{90,000}{2(t+3)^2}\Big|_{28}^{\infty} = \frac{90,000}{2(28+3)^2}$

= 46.8

while Diagram 2 shows that $\sum_{t=28}^{\infty} \frac{90,000}{(t+3)^3} < \int_{27}^{\infty} \frac{90,000}{(t+3)^3} = -\frac{90,000}{2(t+3)^2}\Big|_{28}^{\infty} = \frac{90,000}{2(27+3)^2} = 50.0$

It follows that 46.8 < N = $\sum_{t=28}^{\infty} \frac{90,000}{(t+3)^3} < 50.0$ .

40. D

The marginal distribution of Y is given by $f_2(y) = \int_0^y 6\, e^{-x}\, e^{-2y}\, dx = 6\, e^{-2y} \int_0^y e^{-x}\, dx$

$= -6\, e^{-2y}\, e^{-y} + 6e^{-2y} = 6\, e^{-2y} - 6\, e^{-3y}$, $0 < y < \infty$

Therefore, $E(Y) = \int_0^\infty y\, f_2(y)\, dy = \int_0^\infty (6ye^{-2y} - 6ye^{-3y})\, dy = 6 \int_0^\infty ye^{-2y}\, dy - 6 \int_0^\infty y\, e^{-3y}\, dy$

$= \frac{6}{2}\int_0^\infty 2\, ye^{-2y}\, dy - \frac{6}{3}\int_0^\infty 3\, y\, e^{-3y}\, dy$

But $\int_0^\infty 2\, y\, e^{-2y}\, dy$ and $\int_0^\infty 3\, y\, e^{-3y}\, dy$ are equivalent to the means of exponential random variables with parameters 1/2 and 1/3, respectively. In other words, $\int_0^\infty 2\, y\, e^{-2y}\, dy = 1/2$ and $\int_0^\infty 3\, y\, e^{-3y}\, dy = 1/3$ . We conclude that E(Y) = (6/2) (1/2) – (6/3) (1/3) = 3/2 – 2/3 = 9/6 – 4/6 = 5/6 = 0.83 .