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Mathematical Foundations of Actuarial Science

1

SOCIETY OF ACTUARIES

CASUALTY ACTUARIAL SOCIETY

Wednesday, May 24, 2000

DIRECTIONS

1. DO NOT BREAK THE SEAL OF THE BOOKLET UNTIL THE SUPERVISOR TELLS YOU TO DO SO.

2. This test consists of 40 multiple-choice test questions. You will have a total of 180 minutes in which to answer them and record your answers on the answer sheet. NO ADDITIONAL TIME WILL BE ALLOWED FOR CODING YOUR ANSWER SHEET. Failure to stop writing or coding your answer sheet after time is called will result in the disqualification of your answer sheet and possible further disciplinary action.

3. There are five answer choices for each question, lettered (A) through (E). Answer choices for some questions have been rounded. For each question, choose the best answer. On your answer sheet, find the row of circles with the same number as the question. Then find the circle in that row with the same letter as your answer. Use a soft lead pencil and blacken the circle completely. INDICATE ALL YOUR ANSWERS ON THE ANSWER SHEET. No credit will be given for anything written in the booklet.

Example **Sample Answer**

Calculate the value of x in the equation $x + 6 = -3$.

● Ⓑ Ⓒ Ⓓ Ⓔ

(A) –9
(B) –3
(C) –2
(D) 3
(E) 9

4. Answer sheets are mechanically scored. BE SURE THAT EACH MARK IS BLACK AND COMPLETELY FILLS ONLY THE INTENDED ANSWER CIRCLE. Make no stray marks on the answer sheet. Choose only one answer for each question. If you change an answer, erase your first mark completely and mark your new choice.

5. Use the blank portions of booklet pages for your scratch work. You are not permitted to use extra scratch paper.

6. In questions involving money, no monetary unit is specified, and nothing is implied by the magnitude of the numbers.

7. Do not spend too much time on any question. If a question seems too difficult, go on to the next question. You may return to unanswered questions if you finish before time is called.

8. Your score will be based on the number of questions that you answer correctly, with each question having equal weight. There will be no deduction for wrong answers. It is therefore to your advantage to answer every question.

9. After time is called, the supervisor will collect the booklet and your answer sheet separately. DO NOT ENCLOSE THE ANSWER SHEET IN THE BOOKLET. All booklets and answer sheets must be returned. THE QUESTIONS ARE CONFIDENTIAL AND MAY NOT BE TAKEN FROM THE EXAMINATION ROOM.

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GENERAL INFORMATION

1. $\ln x$ is the natural logarithm of x .

2. $\boldsymbol{R}^n$ is n-dimensional Euclidean space.

3. $\mu_X = E(X)$ denotes the mean of a random variable X .

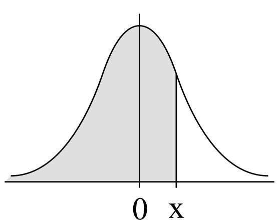
$\sigma_X^2 = Var(X)$ denotes the variance of X .

$\sigma_{XY} = Cov(X,Y)$ denotes the covariance of two random variables X and Y .

$\rho_{XY} = Corr(X,Y)$ denotes the correlation coefficient of X and Y .

$\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$ denotes the mean of a sample $X_1, \ldots, X_n$.

4. The Normal Distribution



The table below gives the value of $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-w^2/2} dw$ for certain values of x . The integer part of x is given in the top row, and the first decimal place of x is given in the left column. Since the density function of x is symmetric, the value of the cumulative distribution function for negative x can be obtained by subtracting from unity the value of the cumulative distribution function for x .

x	0	1	2	3
0.0	0.5000	0.8413	0.9772	0.9987
0.1	0.5398	0.8643	0.9821	0.9990
0.2	0.5793	0.8849	0.9861	0.9993
0.3	0.6179	0.9032	0.9893	0.9995
0.4	0.6554	0.9192	0.9918	0.9997
0.5	0.6915	0.9332	0.9938	0.9998
0.6	0.7257	0.9452	0.9953	0.9998
0.7	0.7580	0.9554	0.9965	0.9999
0.8	0.7881	0.9641	0.9974	0.9999
0.9	0.8159	0.9713	0.9981	1.0000

Selected Points of the Normal Distribution

__(x)	x
0.800	0.842
0.850	1.036
0.900	1.282
0.950	1.645
0.975	1.960
0.990	2.326
0.995	2.576

1. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35% . Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A) 0.05

(B) 0.12

(C) 0.18

(D) 0.25

(E) 0.35

2. A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
1997	0.16	0.05
1998	0.18	0.02
1999	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident.

Determine the probability that the model year of this automobile is 1997 .

(A) 0.22

(B) 0.30

(C) 0.33

(D) 0.45

(E) 0.50

3. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years.

The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year.

If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

(A) 6,321

(B) 7,358

(C) 7,869

(D) 10,256

(E) 12,642

4. Let T denote the time in minutes for a customer service representative to respond to 10 telephone inquiries. T is uniformly distributed on the interval with endpoints 8 minutes and 12 minutes. Let R denote the average rate, in customers per minute, at which the representative responds to inquiries.

Which of the following is the density function of the random variable R on the interval $\left(\frac{10}{12} \le r \le \frac{10}{8}\right)$?

(A) $\frac{12}{5}$

(B) $3 - \frac{5}{2r}$

(C) $3r - \frac{5\ln(r)}{2}$

(D) $\frac{10}{r^2}$

(E) $\frac{5}{2r^2}$

5. Let T_1 and T_2 represent the lifetimes in hours of two linked components in an electronic device. The joint density function for T_1 and T_2 is uniform over the region defined by $0 \le t_1 \le t_2 \le L$ where L is a positive constant.

Determine the expected value of the sum of the squares of T_1 and T_2 .

(A) $\dfrac{L^2}{3}$

(B) $\dfrac{L^2}{2}$

(C) $\dfrac{2L^2}{3}$

(D) $\dfrac{3L^2}{4}$

(E) L^2

6. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

Assuming the two measurements are independent random variables, what is the probability that their average value is within $0.005h$ of the height of the tower?

(A) 0.38

(B) 0.47

(C) 0.68

(D) 0.84

(E) 0.90

7. An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1 + x)^{-4}$, where $0 < x < \infty$.

Determine the company's expected monthly claims.

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 1

(E) 3

8. A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim Size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

What percentage of the claims are within one standard deviation of the mean claim size?

(A) 45%

(B) 55%

(C) 68%

(D) 85%

(E) 100%

9. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{1000} e^{-(x/1000)} \text{ for } x > 0 .$$

The premium for the policy is set at 100 over the expected total claim amount.

If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

(A) 0.001

(B) 0.159

(C) 0.333

(D) 0.407

(E) 0.460

10. An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic Policy claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim is an independent exponential random variable with mean three days.

What is the probability that the next claim will be a Deluxe Policy claim?

(A) 0.172

(B) 0.223

(C) 0.400

(D) 0.487

(E) 0.500

11. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy.

Let X denote the proportion of employees who purchase the basic policy, and Y the proportion of employees who purchase the supplemental policy. Let X and Y have the joint density function $f(x,y) = 2(x + y)$ on the region where the density is positive.

Given that 10% of the employees buy the basic policy, what is the probability that fewer than 5% buy the supplemental policy?

(A) 0.010

(B) 0.013

(C) 0.108

(D) 0.417

(E) 0.500

12. Let C be the curve defined by $x = \sin t + t$ and $y = \cos t - t$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Find an equation of the line tangent to C at $(0, 1)$.

(A) $y = 1$

(B) $y = 1 + 2x$

(C) $y = 1 - 2x$

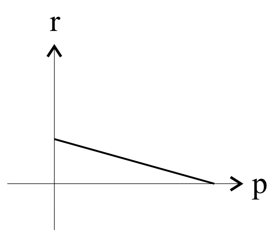
(D) $y = 1 - x$

(E) $y = 1 - \frac{1}{2}x$

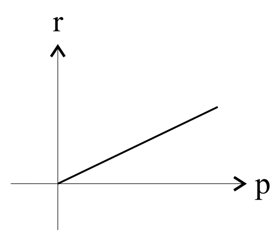
13. For a certain product priced at p per unit, $2000 - 10p$ units will be sold.

Which of the following best represents the graph of revenue, r, as a function of price, p ?

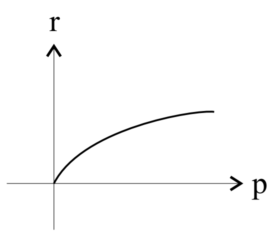
(A)



(B)



(C)



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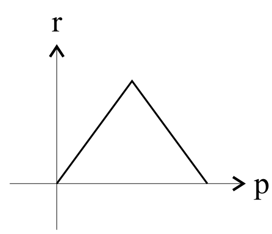
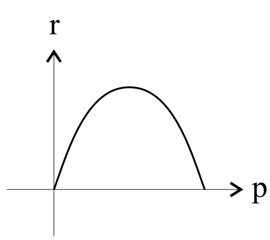


(E)



14. A virus is spreading through a population in a manner that can be modeled by the function

$$g(t) = \frac{A}{1 + Be^{-t}}$$

where A is the total population, $g(t)$ is the number infected at time t, and B is a constant.

What proportion of the population is infected when the virus is spreading the fastest?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

(E) 1

15. In a certain town, the rate of deaths at time t due to a particular disease is modeled by

$$\frac{10{,}000}{(t+3)^3},\ t \geq 0 .$$

What is the total number of deaths from this disease predicted by the model?

(A) 243

(B) 370

(C) 556

(D) 1,111

(E) 10,000

16. The total cost, c, to a company for selling n widgets is $c(n) = n^2 + 4n + 100$. The price per widget is $p(n) = 100 - n$.

What price per widget will yield the maximum profit for the company?

(A) 50

(B) 76

(C) 96

(D) 98

(E) 100

17. An insurance company has 120,000 to spend on the development and promotion of a new insurance policy for car owners. The company estimates that if x is spent on development and y is spent on promotion, then $\frac{x^{1/2}y^{3/2}}{400,000}$ policies will be sold.

Based on this estimate, what is the maximum number of policies that the insurance company can sell?

(A) 3,897

(B) 9,000

(C) 11,691

(D) 30,000

(E) 90,000

18. An insurance policy reimburses dental expense, X, up to a maximum benefit of 250 . The probability density function for X is:

$$f(x) = \begin{cases} ce^{-0.004x} & \text{for } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

Calculate the median benefit for this policy.

(A) 161

(B) 165

(C) 173

(D) 182

(E) 250

19. In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from –2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people.

What is the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages?

(A) 0.14

(B) 0.38

(C) 0.57

(D) 0.77

(E) 0.88

20. Let X and Y denote the values of two stocks at the end of a five-year period. X is uniformly distributed on the interval $(0, 12)$. Given $X = x$, Y is uniformly distributed on the interval $(0, x)$.

Determine $\mathrm{Cov}(X, Y)$ according to this model.

(A) 0

(B) 4

(C) 6

(D) 12

(E) 24

21. A ball rolls along the polar curve defined by $r = \sin\theta$. The ball starts at $\theta = 0$ and ends at $\theta = \frac{3\pi}{4}$.

Calculate the distance the ball travels.

(A) $\frac{3\pi}{8}$

(B) $\frac{3\pi}{4}$

(C) $\frac{9\pi}{8}$

(D) $\frac{3\pi}{2}$

(E) $\frac{15\pi}{8}$

22. An actuary determines that the annual numbers of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number	0	0.12	0.06	0.05	0.02
of tornadoes	1	0.13	0.15	0.12	0.03
in county P	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P .

(A) 0.51

(B) 0.84

(C) 0.88

(D) 0.99

(E) 1.76

23. An insurance policy is written to cover a loss X where X has density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$.

Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A) 0.17

(B) 0.25

(C) 0.32

(D) 0.58

(E) 0.83

24. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

(A) $\frac{1}{\sqrt{3}}$

(B) 1

(C) $\sqrt{2}$

(D) 2

(E) 4

25. An advertising executive claims that, through intensive advertising, 175,000 of a city's 3,500,000 people will recognize the client's product after one day. He further claims that product recognition will grow as advertising continues according to the relationship $a_{n+1} = 0.95a_n + 175{,}000$, where a_n is the number of people who recognize the client's product n days after advertising begins.

If the advertising executive's claims are correct, how many of the city's 3,500,000 people will not recognize the client's product after 35 days of advertising?

(A) 552,227

(B) 561,468

(C) 570,689

(D) 581,292

(E) 611,886

26. The bond yield curve is defined by the function $y(x)$ for $0 < x \le 30$ where y is the yield on a bond which matures in x years. The bond yield curve is a continuous, increasing function of x and, for any two points on the graph of y, the line segment connecting those points lies entirely below the graph of y.

Which of the following functions could represent the bond yield curve?

(A)	$y(x) = a$	a is a positive constant
(B)	$y(x) = a + kx$	a, k are positive constants
(C)	$y(x) = a + k\sqrt{x^3}$	a, k are positive constants
(D)	$y(x) = a + kx^2$	a, k are positive constants
(E)	$y(x) = a + k \log(x + 1)$	a, k are positive constants

27. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P(X = 0, Y = 0) = \frac{1}{6}$$

$$P(X = 1, Y = 0) = \frac{1}{12}$$

$$P(X = 1, Y = 1) = \frac{1}{6}$$

$$\mathrm{P}(X = 2, Y = 0) = \frac{1}{12}$$

$$P(X = 2, Y = 1) = \frac{1}{3}$$

$$P(X = 2, Y = 2) = \frac{1}{6}$$

What is the variance of X ?

(A) 0.47

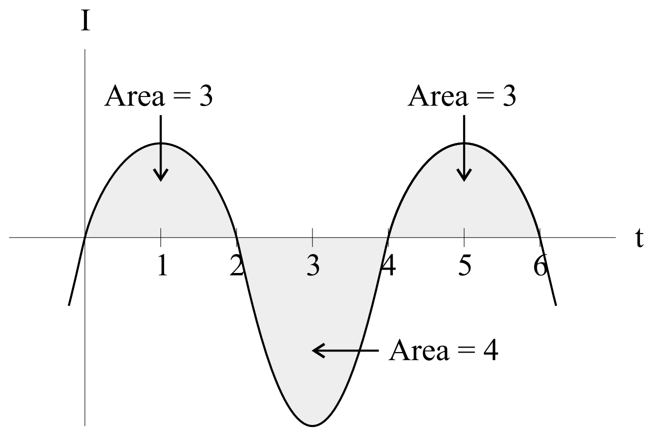
(B) 0.58

(C) 0.83

(D) 1.42

(E) 2.58

28. Inflation is defined as the rate of change in price as a function of time. The figure below is a graph of inflation, I, versus time, t .



Price at time $t = 0$ is 100 .

What is the next time at which price is 100 ?

(A) At some time t, $t \in (0, 2)$.

(B) 2

(C) At some time t, $t \in (2, 4)$.

(D) 4

(E) At some time t, $t \in (4, 6)$.

29. An investor buys one share of stock in an internet company for 100 . During the first four days he owns the stock, the share price changes as follows (measured relative to the prior day's price):

Day 1	Day 2	Day 3	Day 4
up 30%	down 15%	unchanged	down 10%

If the pattern of relative price movements observed on the first four days is repeated indefinitely, how will the price of the share of stock behave in the long run?

(A) It converges to 0.00 .

(B) It converges to 99.45 .

(C) It converges to 101.25 .

(D) It oscillates between two finite values without converging.

(E) It diverges to ∞ .

30. Three radio antennas are located at points (1, 2), (3, 0) and (4, 4) in the *xy*-plane. In order to minimize static, a transmitter should be located at the point which minimizes the sum of the weighted squared distances between the transmitter and each of the antennas. The weights are 5, 10 and 15, respectively, for the three antennas.

What is the *x*-coordinate of the point at which the transmitter should be located in order to minimize static?

(A) 2.67

(B) 3.17

(C) 3.33

(D) 3.50

(E) 4.00

31. Let R be the region bounded by the graph of $x^2 + y^2 = 9$.

Calculate $\iint_R (x^2 + y^2 + 1)\,dA$.

(A) 24π

(B) $\frac{99}{4}\pi$

(C) $\frac{81}{2}\pi$

(D) $\frac{99}{2}\pi$

(E) $\frac{6723}{4}\pi$

32. A study indicates that t years from now the proportion of a population that will be infected with a disease can be modeled by $\mathrm{I}(t) = \dfrac{(t+1)^2}{100}$, $t \le 5$.

Determine the time when the actual proportion infected equals the average proportion infected over the time interval from $t = 0$ to $t = 3$.

(A) 1.38

(B) 1.50

(C) 1.58

(D) 1.65

(E) 1.68

33. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease.

Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

(A) 0.324

(B) 0.657

(C) 0.945

(D) 0.950

(E) 0.995

34. An insurance policy reimburses a loss up to a benefit limit of 10 . The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} \dfrac{2}{y^3} & \text{for } y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0

(B) 1.3

(C) 1.8

(D) 1.9

(E) 2.0

35. A company insures homes in three cities, J, K, and L . Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are independent.

The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1 - 2t)^{-3}$$

$$M_K(t) = (1 - 2t)^{-2.5}$$

$$M_L(t) = (1 - 2t)^{-4.5}$$

Let X represent the combined losses from the three cities.

Calculate $E(X^3)$.

(A) 1,320

(B) 2,082

(C) 5,760

(D) 8,000

(E) 10,560

36. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p_{n+1} = \frac{1}{5} p_n$, where p_n represents the probability that the policyholder files n claims during the period.

Under this assumption, what is the probability that a policyholder files more than one claim during the period?

(A) 0.04

(B) 0.16

(C) 0.20

(D) 0.80

(E) 0.96

37. Let S be the surface described by $f(x,y) = \arctan\left(\dfrac{y}{x}\right)$.

Determine an equation of the plane tangent to S at the point $\left(1,\ 1,\ \dfrac{\pi}{4}\right)$.

(A) $z = \dfrac{\pi}{4} - \dfrac{1}{2}(x-1) - \dfrac{1}{2}(y-1)$

(B) $z = \dfrac{\pi}{4} - \dfrac{1}{2}(x-1) + \dfrac{1}{2}(y-1)$

(C) $z = \dfrac{1}{2}(x-1) + \dfrac{1}{2}(y-1)$

(D) $z = \dfrac{\pi}{4} + \dfrac{1}{2}(x-1) - \dfrac{1}{2}(y-1)$

(E) $z = \dfrac{\pi}{4} + \dfrac{1}{2}(x-1) + \dfrac{1}{2}(y-1)$

38. An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$.

At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250

(B) 375

(C) 500

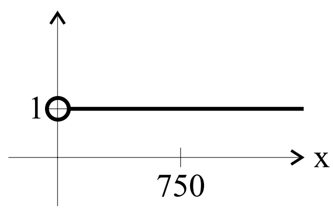
(D) 625

(E) 750

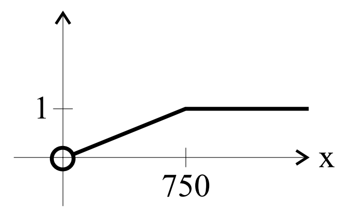
39. An insurance policy is written that reimburses the policyholder for all losses incurred up to a benefit limit of 750 . Let f(x) be the benefit paid on a loss of x .

Which of the following most closely resembles the graph of the derivative of f ?

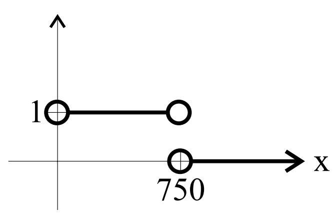
(A)



(B)



(C)



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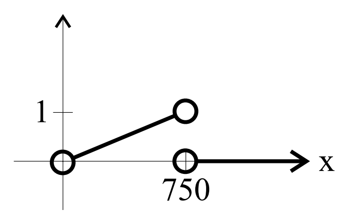

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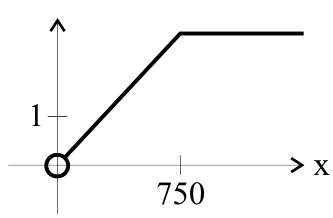


40. A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.

(ii) In any calendar year, the probability of a hurricane is 0.05 .

(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A) 0.06

(B) 0.19

(C) 0.38

(D) 0.62

(E) 0.92

Course 1 May 2000
Answer Key

1.	A	21.	B
2.	D	22.	D
3.	D	23.	A
4.	E	24.	D
5.	C	25.	D
6.	D	26.	E
7.	C	27.	B
8.	A	28.	C
9.	B	29.	A
10.	C	30.	B
11.	D	31.	D
12.	E	32.	D
13.	E	33.	B
14.	B	34.	D
15.	C	35.	E
16.	B	36.	A
17.	C	37.	B
18.	C	38.	C
19.	D	39.	C
20.	C	40.	E

1. Answer: A

   Let R = event of referral to a specialist

   L = event of lab work

   We want to find

   $P[R\cap L] = P[R] + P[L] - P[R\cup L] = P[R] + P[L] - 1 + P[\sim(R\cup L)]$

   $= P[R] + P[L] - 1 + P[\sim R\cap\sim L] = 0.30 + 0.40 - 1 + 0.35 = 0.05$.

2. Answer: D

   Use Baye's Theorem with A = the event of an accident in one of the years 1997, 1998 or 1999 .

   $$P[1997|A] = \frac{P[A|1997]P[1997]}{P[A|1997][P[1997]+P[A|1998]P[1998]+P[A|1999]P[1999]}$$

   $$= \frac{(0.05)(0.16)}{(0.05)(0.16)+(0.02)(0.18)+(0.03)(0.20)} = 0.45 .$$

3. Answer: D

   Let T denote printer lifetime. Then $f(t) = \frac{1}{2} e^{-t/2}$, $0 \le t \le \infty$

   Note that

   $$P[T \le 1] = \int_0^1 \frac{1}{2}e^{-t/2}dt = e^{-t/2}\Big|_0^1 = 1 - e^{-1/2} = 0.393$$

   $$P[1 \le T \le 2] = \int_1^2 \frac{1}{2}e^{-t/2}dt = e^{-t/2}\Big|_1^2 = e^{-1/2} - e^{-1} = 0.239$$

   Next, denote refunds for the 100 printers sold by independent and identically distributed random variables $Y_1, \ldots, Y_{100}$ where

   $$Y_i = \begin{cases} 200 & \text{with probability } 0.393 \\ 100 & \text{with probability } 0.239 \\ 0 & \text{with probability } 0.368 \end{cases} \qquad i = 1, \ldots, 100$$

   Now $E[Y_i] = 200(0.393) + 100(0.239) = 102.56$

   Therefore, Expected Refunds $= \sum_{i=1}^{100} E[Y_i] = 100(102.56) = 10{,}256$.

4. Answer: E

First note R = 10/T . Then

$F_R(r) = P[R \le r] = P\left[\frac{10}{T} \le r\right] = P\left[T \ge \frac{10}{r}\right] = 1 - F_T\left(\frac{10}{r}\right)$. Differentiating with respect to r

$f_R(r) = F'_R(r) = d/dr\left(1 - F_T\left(\frac{10}{r}\right)\right) = -\left(\frac{d}{dt}F_T(t)\right)\left(\frac{-10}{r^2}\right)$

$\frac{d}{dt}F_T(t) = f_T(t) = \frac{1}{4}$ since T is uniformly distributed on [8, 12] .

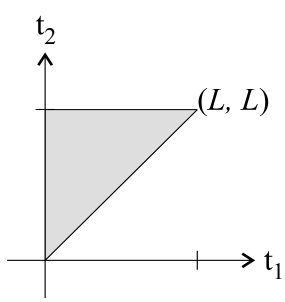
Therefore $f_R(r) = \frac{-1}{4}\left(\frac{-10}{r^2}\right) = \frac{5}{2r^2}$.

5. Answer: C

We are given $f(t_1, t_2) = 2/L^2$, $0 \le t_1 \le t_2 \le L$.

Therefore, $E[T_1^2 + T_2^2] = \int_0^L\int_0^{t_2}(t_1^{\,2} + t_2^{\,2})\frac{2}{L^2}dt_1dt_2 = \frac{2}{L^2}\left\{\int_0^L\left[\frac{t_1^{\,3}}{3} + t_2^{\,2}t_1\right]_0^{t_2} dt_1\right\} = \frac{2}{L^2}\left\{\int_0^L\left(\frac{t_2^{\,3}}{3} + t_2^{\,3}\right)dt_2\right\}$

$= \frac{2}{L^2}\int_0^L \frac{4}{3}t_2^{\,3}dt_2 = \frac{2}{L^2}\left[\frac{t_2^{\,4}}{3}\right]_0^L = \frac{2}{3}L^2$



6. Answer: D

Let X_1 and X_2 denote the measurement errors of the less and more accurate instruments, respectively. If $N(\mu,\sigma)$ denotes a normal random variable with mean μ and standard deviation σ, then we are given X_1 is N(0, 0.0056h), X_2 is N(0, 0.0044h) and X_1, X_2 are independent. It follows that $Y = \frac{X_1 + X_2}{2}$ is $N\left(0, \sqrt{\frac{0.0056^2h^2 + 0.0044^2h^2}{4}}\right)$ = N(0, 0.00356h) . Therefore,

$P[-0.005h \le Y \le 0.005h] = P[Y \le 0.005h] - P[Y \le -0.005h] = P[Y \le 0.005h] - P[Y \ge 0.005h]$

$= 2P[Y \le 0.005h] - 1 = 2P\left[Z \le \frac{0.005h}{0.00356h}\right] - 1 = 2P[Z \le 1.4] - 1 = 2(0.9192) - 1 = 0.84$.

7. Answer: C

The pdf of x is given by f(x) = $\dfrac{k}{(1+x)^4}$, $0 < x < \infty$. To find k, note

$$1 = \int_0^\infty \frac{k}{(1+x)^4}dx = -\frac{k}{3}\frac{1}{(1+x)^3}\Big|_0^\infty = \frac{k}{3}$$

k = 3

It then follows that E[x] = $\int_0^\infty \frac{3x}{(1+x)^4}dx$ and substituting u = 1 + x, du = dx, we see

$$E[x] = \int_1^\infty \frac{3(u-1)}{u^4}du = 3\int_1^\infty (u^{-3}-u^{-4})du = 3\left[\frac{u^{-2}}{-2}-\frac{u^{-3}}{-3}\right]_1^\infty = 3\left[\frac{1}{2}-\frac{1}{3}\right] = 3/2 - 1 = ½ .$$

8. Answer: A

Let X denote claim size. Then E[X] = [20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30)] = (3 + 3 + 2 + 10 + 6 + 7 + 24) = 55

$E[X^2]$ = 400(0.15) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30) = 60 + 90 + 80 + 500 + 360 + 490 + 1920 = 3500

Var[X] = $E[X^2] - (E[X])^2$ = 3500 – 3025 = 475 and $\sqrt{Var[X]}$ = 21.79 .

Now the range of claims within one standard deviation of the mean is given by [55.00 – 21.79, 55.00 + 21.79] = [33.21, 76.79]

Therefore, the proportion of claims within one standard deviation is 0.05 + 0.20 + 0.10 + 0.10 = 0.45 .

9. Answer: B

Denote the policy premium by P . Since x is exponential with parameter 1000, it follows from what we are given that E[X] = 1000, Var[X] = 1,000,000, $\sqrt{Var[X]}$ = 1000 and P = 100 + E[X] = 1,100 . Now if 100 policies are sold, then Total Premium Collected = 100(1,100) = 110,000

Moreover, if we denote total claims by S, and assume the claims of each policy are independent of the others then E[S] = 100 E[X] = (100)(1000) and Var[S] = 100 Var[X] = (100)(1,000,000) .

It follows from the Central Limit Theorem that S is approximately normally distributed with mean 100,000 and standard deviation = 10,000 . Therefore, P[S ≥ 110,000] = 1 – P[S ≤ 110,000]

$= 1 - P\left[Z \le \frac{110,000-100,000}{10,000}\right]$ = 1 – P[Z ≤ 1] = 1 – 0.841 ≈ 0.159 .

10. Answer: C

Let T_1 be the time until the next Basic Policy claim, and let T_2 be the time until the next Deluxe policy claim. Then the joint pdf of T_1 and T_2 is

$f(t_1,t_2) = \left(\frac{1}{2}e^{-t_1/2}\right)\left(\frac{1}{3}e^{-t_2/3}\right) = \frac{1}{6}e^{-t_1/2}e^{-t_2/3}$, $0 < t_1 < \infty$, $0 < t_2 < \infty$ and we need to find

$$P[T_2 < T_1] = \int_0^{\infty}\int_0^{t_1} \frac{1}{6}e^{-t_1/2}e^{-t_2/3}dt_2 dt_1 = \int_0^{\infty}\left[-\frac{1}{2}e^{-t_1/2}e^{-t_2/3}\right]_0^{t_1} dt_1$$

$$= \int_0^{\infty}\left[\frac{1}{2}e^{-t_1/2} - \frac{1}{2}e^{-t_1/2}e^{-t_1/3}\right]dt_1 = \int_0^{\infty}\left[\frac{1}{2}e^{-t_1/2} - \frac{1}{2}e^{-5t_1/6}\right]dt_1 = \left[-e^{-t_1/2} + \frac{3}{5}e^{-5t_1/6}\right]_0^{\infty} = 1 - \frac{3}{5} = \frac{2}{5} = 0.4 .$$

11. Answer: D

We are given that the joint pdf of X and Y is $f(x,y) = 2(x+y)$, $0 < y < x < 1$.

Now $f_x(x) = \int_0^x (2x+2y)dy = \left[2xy + y^2\right]_0^x = 2x^2 + x^2 = 3x^2$, $0 < x < 1$

so $f(y|x) = \frac{f(x,y)}{f_x(x)} = \frac{2(x+y)}{3x^2} = \frac{2}{3}\left(\frac{1}{x} + \frac{y}{x^2}\right)$, $0 < y < x$

$f(y|x = 0.10) = \frac{2}{3}\left[\frac{1}{0.1} + \frac{y}{0.01}\right] = \frac{2}{3}\left[10 + 100y\right]$, $0 < y < 0.10$

$P[Y < 0.05|X = 0.10] = \int_0^{0.05} \frac{2}{3}\left[10 + 100y\right]dy = \left[\frac{20}{3}y + \frac{100}{3}y^2\right]_0^{0.05} = \frac{1}{3} + \frac{1}{12} = \frac{5}{12} = 0.4167$.

12. Answer: E

We are given $x = \sin(t) + t$, $y = \cos(t) - t$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. We want to find the slope of the tangent line at $(x,y) = (0,1)$. Therefore, note $0 = \sin(t) + t \Rightarrow \sin(t) = -t \Rightarrow t = 0$. Then $dx/dt = \cos(t) + 1$, $dy/dt = -\sin(t) - 1$. Then $\left.\frac{dy}{dx}\right|_{(x,y)=(0,1)} = \left.\frac{dy}{dt} / \frac{dx}{dt}\right|_{t=0} = \frac{-\sin(0) - 1}{\cos(0) + 1} = -\frac{1}{2}$.

The equation of the tangent line is given by $y - 1 = (-1/2)(x - 0)$ or $y = (-1/2)x + 1$.

13. Answer: E

$r(p) = (2000 - 10p)p = 2000p - 10p^2$. The graph of this function hits the x axis twice (at $p = 0$ and $p = 200$). The derivative $r'(p) = 2000 - 20p$ implies that the graph in E is better than the graph in D .

14. Answer: B

The number of persons infected by a virus is modeled by g(t) = $\frac{A}{1+Be^{-t}}$, t > 0 . It follows that the rate at which persons are infected is given by g′(t) = $\frac{ABe^{-t}}{(1+Be^{-t})^2}$, t > 0 . We need to maximize g′(t) . Considering that the maximum of g′(t) will occur at t such that g″(t) = 0, take

$$g''(t) = \frac{-ABe^{-t}(1+Be^{-t})^2+2AB^2e^{-2t}(1+Be^{-t})}{(1+Be^{-t})^4} = \frac{ABe^{-t}[-(1+Be^{-t})+2Be^{-t}]}{(1+Be^{-t})^3} = \frac{ABe^{-t}(Be^{-t}-1)}{(1+Be^{-t})^3}$$

= 0 . It follows that g′(t) is maximized when $Be^{-t} - 1 = 0$ or $Be^{-t} = 1$. This depends on g″(t) going from > 0 to < 0 as t increases through the critical point. Therefore, at the maximum

$$g(t) = \frac{A}{1+Be^{-t}} = \frac{A}{1+1} = \frac{1}{2}A$$

15. Answer: C

$$\text{Total deaths} = \int_0^\infty \frac{10,000}{(t+3)^3}dt = \left.\frac{-10,000}{2(t+3)^2}\right|_0^\infty = \frac{10,000}{2(3)^2} = \frac{10,000}{18} = 556 .$$

16. Answer: B

Denote profit by P(n) . Then

P(n) = n × p(n) – c(n)

= n(100 – n) – n^2 – 4n – 100

= 100n – n^2 – n^2 – 4n – 100

= –2n^2 + 96n – 100

Then P′(n) = –4n + 96 = 0 ⇒ 4n = 96 ⇒ n = 24 and the profit-maximizing price is p(24) = 100 – 24 = 76 .

17. Answer: C

The company's spending is constrained to x + y = 120,000 $\Rightarrow$ y = 120,000 – x . Therefore, we want to find the maximum of the function f(x) = $\dfrac{x^{1/2}(120,000-x)^{3/2}}{400,000}$, $0 \le x \le 120,000$

$$f'(x) = \frac{1}{400,000}\left[\frac{1}{2}x^{-1/2}(120,000-x)^{3/2} - \frac{3}{2}x^{1/2}(120,000-x)^{1/2}\right]$$

$$= \frac{1}{2(400,000)}x^{-1/2}\ (120,000-x)^{1/2}\ [120,000-4x] = 0\ .$$

It follows that x = 30,000 or x = 120,000

Since f(120,000) = 0, the maximum number of policies is given by

$$f(30,000) = \frac{(30,000)^{1/2}(90,000)^{3/2}}{400,000} = 11,691\ .$$

Alternate Solution using Lagrange Multipliers

Maximize $\dfrac{x^{1/2}y^{3/2}}{400,000}$ subject to x + y – 120,000 = 0

Using Lagrange multipliers

$$\frac{\partial}{\partial x}\frac{x^{1/2}y^{3/2}}{400,000} = \lambda\frac{\partial}{\partial x}(x + y - 120,000)$$

$$\frac{\partial}{\partial y}\frac{x^{1/2}y^{3/2}}{400,000} = \lambda\frac{\partial}{\partial y}(x + y - 120,000)$$

$$\left.\begin{aligned}\left(\frac{1}{2}\right)\frac{x^{-1/2}y^{3/2}}{400,000} &= \lambda\cdot 1\\ \left(\frac{3}{2}\right)\frac{x^{1/2}y^{1/2}}{400,000} &= \lambda\cdot 1\end{aligned}\right\}\Rightarrow \begin{aligned}x^{-1/2}y^{3/2} &= 3x^{1/2}y^{1/2}\\ y &= 3x\end{aligned}$$

Then x + y = 120,000 and y = 3x gives 4x = 120,000

x = 30,000

y = 90,000

Maximum number of policies = $\dfrac{(30,000)^{1/2}(90,000)^{3/2}}{400,000}$ = 11,691 .

18. Answer: C

Note that X has an exponential distribution. Therefore, c = 0.004 . Now let Y denote the claim benefits paid. Then $Y = \begin{cases} x & \text{for } x < 250 \\ 250 & \text{for } x \geq 250 \end{cases}$ and we want to find m such that

$$0.50 = \int_0^m 0.004e^{-0.004x}dx = -e^{-0.004x}\Big|_0^m = 1 - e^{-0.004m}$$

This condition implies $e^{-0.004m} = 0.5 \Rightarrow m = 250 \ln 2 = 173.29$.

19. Answer: D

Let X denote the difference between true and reported age. We are given X is uniformly distributed on $(-2.5, 2.5)$. That is, X has pdf f(x) = 1/5, $-2.5 < x < 2.5$. It follows that

μ_x = E[X] = 0

$$\sigma_x^2 = \text{Var}[X] = E[X^2] = \int_{-2.5}^{2.5} \frac{x^2}{5}dx = \frac{x^3}{15}\Big|_{-2.5}^{2.5} = \frac{2(2.5)^3}{15} = 2.083$$

$\sigma_x = 1.443$

Now $\bar{X}_{48}$, the difference between the means of the true and rounded ages, has a distribution that is approximately normal with mean 0 and standard deviation $\frac{1.443}{\sqrt{48}} = 0.2083$. Therefore,

$$P\left[-\frac{1}{4} \leq \bar{X}_{48} \leq \frac{1}{4}\right] = P\left[\frac{-0.25}{0.2083} \leq Z \leq \frac{0.25}{0.2083}\right] = P[-1.2 \leq Z \leq 1.2] = P[Z \leq 1.2] - P[Z \leq -1.2]$$

$$= P[Z \leq 1.2] - 1 + P[Z \leq 1.2] = 2P[Z \leq 1.2] - 1 = 2(0.8849) - 1 = 0.77 .$$

20. Answer: C

The joint pdf of X and Y is $f(x,y) = f_2(y|x)\, f_1(x) = (1/x)(1/12)$, $0 < y < x$, $0 < x < 12$.

Therefore,

$$E[X] = \int_0^{12}\int_0^x x \cdot \frac{1}{12x}dydx = \int_0^{12} \frac{y}{12}\Big|_0^x dx = \int_0^{12} \frac{x}{12}dx = \frac{x^2}{24}\Big|_0^{12} = 6$$

$$E[Y] = \int_0^{12}\int_0^x \frac{y}{12x}dydx = \int_0^{12}\left[\frac{y^2}{24x}\right]_0^x dx = \int_0^{12} \frac{x}{24}dx = \frac{x^2}{48}\Big|_0^{12} = \frac{144}{48} = 3$$

$$E[XY] = \int_0^{12}\int_0^x \frac{y}{12}dydx = \int_0^{12}\left[\frac{y^2}{24}\right]_0^x dx = \int_0^{12} \frac{x^2}{24}dx = \frac{x^3}{72}\Big|_0^{12} = \frac{(12)^3}{72} = 24$$

Cov(X,Y) = E[XY] – E[X]E[Y] = 24 – (3)(6) = 24 – 18 = 6 .

21. Answer: B

Denote arc length by L . Then L = $\int_0^{3\pi/4} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ where $r = \sin\theta, \frac{dr}{d\theta} = \cos\theta$

$= \int_0^{3\pi/4} \sqrt{\sin^2\theta + \cos^2\theta}\, d\theta = \int_0^{3\pi/4} d\theta = \theta \Big|_0^{3\pi/4} = \frac{3\pi}{4}$

22. Answer: D

Denote the number of tornadoes in counties P and Q by N_P and N_Q, respectively. Then

$E[N_Q|N_P = 0] = [(0)(0.12) + (1)(0.06) + (2)(0.05) + 3(0.02)] / [0.12 + 0.06 + 0.05 + 0.02] = 0.88$

$E[N_Q^2|N_P = 0] = [(0)^2(0.12) + (1)^2(0.06) + (2)^2(0.05) + (3)^2(0.02)] / [0.12 + 0.06 + 0.05 + 0.02]$

$= 1.76$ and $Var[N_Q|N_P = 0] = E[N_Q^2|N_P = 0] - \{E[N_Q|N_P = 0]\}^2 = 1.76 - (0.88)^2 = 0.9856$.

23. Answer: A

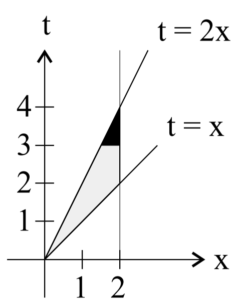
We are given that X denotes loss. In addition, denote the time required to process a claim by T .

Then the joint pdf of X and T is $f(x,t) = \begin{cases} \frac{3}{8}x^2 \cdot \frac{1}{x} = \frac{3}{8}x, & x < t < 2x, 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$

Now we can find P[T ≥ 3] =

$\int_3^4 \int_{t/2}^2 \frac{3}{8} x\,dx\,dt = \int_3^4 \left[\frac{3}{16}x^2\right]_{t/2}^2 dt = \int_3^4 \left(\frac{12}{16} - \frac{3}{64}t^2\right) dt = \left[\frac{12}{16} - \frac{1}{64}t^3\right]_3^4 = \frac{12}{4} - 1 - \left(\frac{36}{16} - \frac{27}{64}\right)$

= 11/64 = 0.17 .



24. Answer: D

Let N be the number of claims filed. We are given P[N = 2] = $\frac{e^{-\lambda}\lambda^2}{2!} = 3\frac{e^{-\lambda}\lambda^4}{4!}$ = 3 · P[N = 4]

$24\lambda^2 = 6\lambda^4$

$\lambda^2 = 4 \Rightarrow \lambda = 2$

Therefore, Var[N] = λ = 2 .

25. Answer: D

We are looking for 3,500,000 – a_{35} .

a_1 = 175,000

$a_2 = 0.95a_1 + 175{,}000 = 0.95(175{,}000) + 175{,}000$

$a_3 = 0.95a_2 + 175{,}000 = 0.95^2(175{,}000) + 0.95(175{,}000) + 175{,}000$

$\vdots$

$a_{35} = 0.95^{34}(175{,}000) + 0.95^{33}(175{,}000) + \ldots + 175{,}000 = 175{,}000(0.95^{34} + 0.95^{33} + \ldots +1)$

$= 175{,}000\left(\dfrac{1-0.95^{35}}{1-0.95}\right) = 2{,}918{,}708$

Answer = 3,500,000 – 2,918,708 = 581,292 .

26. Answer: E

Since y(x) is increasing and from the second condition $y'(x)$ is decreasing, we know $y'(x) > 0$ and $y''(x) < 0$. Note that $y(x) = a + k\log(x + 1)$ works because $y'(x) = k/(x+1) > 0$ and $y''(x) = -k/((x+1)^2) < 0$ for $k > 0$, $0 < x \leq 30$.

27. Answer: B

Note

$P(X = 0) = 1/6$

$P(X = 1) = 1/12 + 1/6 = 3/12$

$P(X = 2) = 1/12 + 1/3 + 1/6 = 7/12$.

$E[X] = (0)(1/6) + (1)(3/12) + (2)(7/12) = 17/12$

$E[X^2] = (0)^2(1/6) + (1)^2(3/12) + (2)^2(7/12) = 31/12$

$Var[X] = 31/12 - (17/12)^2 = 0.58$.

28. Answer: C

Let I = inflation and P = price. Given 1 = dP/dt and P(0) = 100 it follows that $P = 100 + \int_0^t I(u)du$. The next time P = 100 is when $\int_0^t I(u)du = 0$ which occurs when the area above the x axis minus the area below the x axis is 0 .

29. Answer: A

Let P_n be the price of the stock on day n . Then

$P_{4k} = 100\,(1.3)^k(0.85)^k(1.00)^k(0.90)^k = 100(0.9945)^k$

$P_{4k+1} = 100\,(1.3)^{k+1}(0.85)^k(1.00)^k(0.90)^k = 130\,(0.9945)^k$

$P_{4k+2} = 100\,(1.3)^{k+1}(0.85)^{k+1}(1.00)^k(0.90)^k = 110.5\,(0.9945)^k$

$P_{4k+3} = 100\,(1.3)^{k+1}(0.85)^{k+1}(1.00)^{k+1}(0.90)^k = 110.5\,(0.9945)^k$

Since 0.9945 < 1, it follows that $\lim\limits_{k\to\infty} P_{4k} = \lim\limits_{k\to\infty} P_{4k+1} = \lim\limits_{k\to\infty} P_{4k+2} = \lim\limits_{k\to\infty} P_{4k+3} = 0$.

We conclude that $\lim\limits_{n\to\infty} P_n = 0$.

30. Answer: B

The function to be minimized is

$f(x,y) = 5[(x-1)^2 + (y-2)^2] + 10[(x-3)^2 + (y-0)^2] + 15[(x-4)^2 + (y-4)^2]$

This will occur when both

$g(x) = 5(x-1)^2 + 10(x-3)^2 + 15(x-4)^2$ and $h(y) = 5(y-2)^2 + 10y^2 + 15(y-4)^2$ are minimized.

Setting $g'(x) = 10(x-1) + 20(x-3) + 30(x-4) = 0$ determines x .

$10x - 10 + 20x - 60 + 30x - 120 = 0$

$60x = 190$

$x = 190/6 = 3.17$.

31. Answer: D

$$\iint_R (x^2+y^2+1)\,dA = \iint_R (r^2\cos^2\theta + r^2\sin^2\theta + 1)\,dA$$

$$= \int_0^{2\pi}\int_0^3 (r^2+1)r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 (r^3+r)\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^4}{4}+\frac{r^2}{2}\right]_0^3 d\theta = \int_0^{2\pi}\left(\frac{81}{4}+\frac{9}{2}\right)d\theta = \frac{99}{4}\int_0^{2\pi} d\theta = \frac{99}{2}\pi$$

32. Answer: D

The average fraction over the time interval [0,3] is given by

$$A = \frac{1}{3}\int_0^3 \frac{(t+1)^2}{100}\,dt = \frac{1}{3}\,\frac{(t+1)^3}{300}\Bigg|_0^3 = \frac{64}{900} - \frac{1}{900} = \frac{63}{900} = \frac{7}{100}$$

Therefore, the time T when A = I(t) is given by $7/100 = ((T+1)^2)/100 \Rightarrow T = \sqrt{7} - 1 = 1.65$.

33. Answer: B

Let Y = positive test result

D = disease is present (and ~D = not D)

Using Baye's theorem:

$$P[D|Y] = \frac{P[Y \mid D]P[D]}{P[Y \mid D]P[D] + P[Y \mid \sim D]P[\sim D]} = \frac{(0.95)(0.01)}{(0.95)(0.01) + (0.005)(0.99)} = 0.657 .$$

34. Answer: D

Let W denote claim payments. Then $W = \begin{cases} y & \text{for } 1 < y \le 10 \\ 10 & \text{for } y \ge 10 \end{cases}$

It follows that $E[W] = \int_1^{10} y \frac{2}{y^3} dy + \int_{10}^{\infty} 10 \frac{2}{y^3} dy = -\frac{2}{y}\Big|_1^{10} - \frac{10}{y^2}\Big|_{10}^{\infty} = 2 - 2/10 + 1/10 = 1.9$.

35. Answer: E

Let X_J, X_K, and X_L represent annual losses for cities J, K, and L, respectively. Then $X = X_J + X_K + X_L$ and due to independence

$M(t) = E\left[e^{xt}\right] = E\left[e^{(x_J + x_K + x_L)t}\right] = E\left[e^{x_J t}\right]E\left[e^{x_K t}\right]E\left[e^{x_L t}\right]$

$= M_J(t)\, M_K(t)\, M_L(t) = (1 - 2t)^{-3}\, (1 - 2t)^{-2.5}\, (1 - 2t)^{-4.5} = (1 - 2t)^{-10}$

Therefore,

$M'(t) = 20(1 - 2t)^{-11}$

$M''(t) = 440(1 - 2t)^{-12}$

$M'''(t) = 10{,}560(1 - 2t)^{-13}$

$E[X^3] = M'''(0) = 10{,}560$

36. Answer: A

$$p_k = \frac{1}{5} p_{k-1} = \frac{1}{5}\frac{1}{5} p_{k-2} = \frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5} p_{k-3} = \dots = \left(\frac{1}{5}\right)^k p_0 \quad k \ge 0$$

$$1 = \sum_{k=0}^{\infty} p_k = \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k p_0 = \frac{p_0}{1 - \frac{1}{5}} = \frac{5}{4} p_0$$

$p_0 = 4/5$.

Therefore, $P[N > 1] = 1 - P[N \le 1] = 1 - (4/5 + 4/5 \cdot 1/5) = 1 - 24/25 = 1/25 = 0.04$.

37. Answer: B

f(x,y) = arctan (y/x)

$f_x(x,y) = \dfrac{1}{1+\left(\dfrac{y}{x}\right)^2}\left(-\dfrac{y}{x^2}\right) \Rightarrow f_x(1,1) = -\tfrac{1}{2}$

$f_y(x,y) = \dfrac{1}{1+\left(\dfrac{y}{x}\right)^2}\left(\dfrac{1}{x}\right) \Rightarrow f_y(1,1) = \tfrac{1}{2}$

$(z - \pi/4) - \tfrac{1}{2}(x - 1) + \tfrac{1}{2}(y - 1) = 0$

$z = \pi/4 - \tfrac{1}{2}(x - 1) + \tfrac{1}{2}(y - 1)$.

38. Answer: C

Let Y represent the payment made to the policyholder for a loss subject to a deductible D . That is $Y = \begin{cases} 0 & \text{for } 0 \le X \le D \\ x - D & \text{for } D < X \le 1 \end{cases}$

Then since E[X] = 500, we want to choose D so that

$$\frac{1}{4}500 = \int_D^{1000} \frac{1}{1000}(x-D)dx = \frac{1}{1000}\frac{(x-D)^2}{2}\Bigg|_D^{1000} = \frac{(1000-D)^2}{2000}$$

$(1000 - D)^2 = 2000/4 \cdot 500 = 500^2$

$1000 - D = \pm 500$

D = 500 (or D = 1500 which is extraneous).

39. Answer: C

$$f(x) = \begin{cases} x & \text{for } 0 < x \le 750 \\ 750 & \text{for } x > 750 \end{cases} \Rightarrow f'(x) = \begin{cases} 1 & \text{for } 0 < x < 750 \\ 0 & \text{for } x > 750 \end{cases}$$

40. Answer: E

Let X be the number of hurricanes over the 20-year period. The conditions of the problem give x is a binomial distribution with n = 20 and p = 0.05 . It follows that

$P[X < 2] = (0.95)^{20}(0.05)^0 + 20(0.95)^{19}(0.05) + 190(0.95)^{18}(0.05)^2$

$= 0.358 + 0.377 + 0.189 = 0.925$.