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A Computational Model of Mars Craters-Size Frequency Distribution

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Abstract

In the following paper a computational model is presented that reproduces essential features (slopes) of the crater-size frequency distribution (CSFD) of the 42,283 craters contained in the Barlow's Mars Catalog. The model, based on the Monte Carlo method, assumes that the Martian surface is a two dimensional grid and simulates the crater formation process due to impacts, of different diameters, falling uniformly at random locations. The diameters of the simulated impacts were randomly generated from a power law distribution (α=4.3) where smaller impacts are more frequent than larger impacts. The simulation considers the elimination of craters by the obliteration of smaller craters due to large impacts. The model provides a simple and natural explanation for the presence of different slopes in the observed log-log plot of numbers (N) vs. diameter (D).

1. Introduction

Besides the major planets, the solar system contains a large number of smaller bodies. The asteroids and the comets are examples of smaller bodies orbiting the solar system with short and long periodic orbits. In many cases the asteroids and comets may leave their orbits and either strike the Sun or be ejected of the solar system. In other cases they may strike the surface of planetary bodies and leave a record of craters in its surface. This record of impact provides valuable information about the evolution of the solar system and its planetary surfaces. The number of craters is typically analyzed by counting the number of craters in a certain area (crater density) or by counting the number of craters of a certain diameter (crater-size frequency distribution-CSFD). A higher *crater density* implies an older planetary surface and the CSFD may be used to estimate the probability of impacts due to asteroids/comets of a certain size [1].

A simple mathematical model for the CSFD assumes a power-law distribution given by:

$$N \propto D^{-\alpha} \tag{1}$$

where N is the number of craters of a certain diameter D, and α is the slope in a log-log plot. The negative sign in the slope indicates that small craters are very numerous meanwhile bigger craters are less-numerous. In general the value of α is different for the planetary surface under consideration and even for the same planetary surface we can observe breaks in the power-law. The observed slopes in the CSFD are related with two physical processes that are important in the formation of craters. The first process is the erosion of the surface and the second is the obliteration or erasing of craters due to subsequent impacts. One of the main areas of research in planetary sciences requires the interpretation and analysis of the CSFD in order to identify the main process occurring in the surface. Several analytical and numerical techniques have been proposed for this analysis [7].

Among the planetary systems around us Mars is of particular interest due to the closeness to the Earth and the understanding of its impact processes may give important clues about this activity in the Earth. This issue may be crucial to understand and elucidate several hypotheses about periods of heavy asteroids/comets bombardment in the Earth and its relation with the extinctions of Dinosaurs.

In this paper a computational model is presented to explain the observed CSFD in Mars. For craters in the range from 20 to 50 Km, the computational model simulates the formation of craters in the surface of Mars by considering the obliteration as the main process. In the next section the main assumptions of the model and the parameters are presented, including the implementation of the model and finally some results.

2. Data

The Mars craters impact data was obtained from Nadine Barlow's *Catalog of Large Martians Craters* [2], which is considered to be the most complete data set. It contains 42,283 impact craters distributed globally as measured off by the Viking 1:2,000,000 photo mosaic maps.

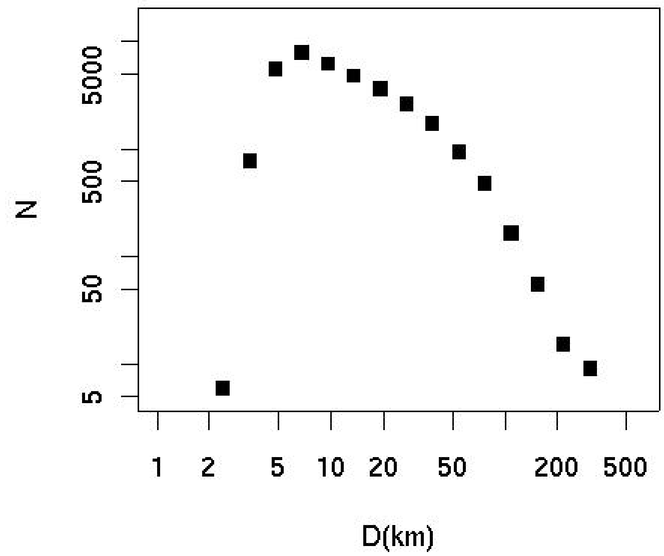

Figure 1. Crater size frequency distribution for the 42,283 impact craters in Nadine Barlow's Catalog.

The database contains craters in the range from 1 to more than 600 km. The CSFD for this catalog is shown in Figure 1. From the observed CSFD we can identify several regions of interest: for craters smaller than 5 km we observe that the number decays dramatically, for craters in the range from 5 to 50 Km we observe a slope $\alpha = 1.8$ (flatter), and for craters above 50 km we observe a slope $\alpha = 4.3$ (stepper). Our hypothesis for this article is that the stepper slope is due to the non-obliterated craters and the flatter slope is related to the obliteration process. We want to test this hypothesis by computer simulations.

3. Computational Method

The computational model used in this work is based in the Mote Carlo method [3]. The Monte Carlo methods are a class of computational algorithms for simulating behavior. They are distinguished from other simulation methods (such as molecular dynamics) for being nondeterministic in some manner - usually using random numbers (or more often pseudo-random numbers) - as opposed to deterministic algorithms. These methods has been used for centuries, but in a few past decades has been introduced to the computational models, because of the ability of the computers to manage random numbers. A classic use of this method is the evaluation of definite integrals, particularly multidimensional integrals with complicated boundary conditions.

The implementation of the Monte Carlo method in this work assumes that the impacts diameters (circles) are randomly generated from a power-law distribution with $\alpha = 4.3$, which is consistent with the observed CSFD for Mars. Once the impacts are generated, the locations in the grid are determined from uniformly random numbers for the vertical and horizontal coordinates. The repeated superposition of "circles" results in the obliteration of smaller "craters" due to large impacts. The "surviving" craters were analyzed by constructing the CSFD.

3.1. Algorithm

- Define a rectangular grid to simulate the surface of Mars.
- Generate N artificial impacts with random (power-law distribution) diameters at uniformly random positions in the surface.
- The impacts produce circular features (craters).
- If a bigger impact falls above a smaller crater and, is covered completely, the smaller crater is erased.
- The accumulation of smaller impacts, in a bigger crater, may result in the erasing of bigger craters if it's smaller than a critical-diameter (e.g. 50 Km).
- The simulated impacted surface is analyzed by constructing a log-log crater-size frequency distribution.

3.2 Implementation

The model was developed using the modeling environment NetLogo developed by Uri Wilensky, at Northwestern University's Center for Connected Learning and Computer-Based Modeling. This language has the capability to program hundreds or thousands of "agents" all operating concurrently. It can be used to model natural and social problems, and is a powerful tool in research.

The usage of NetLogo is very simple. First you define an agent (called generically a `turtle`) in an environment where you determine the rules. A NetLogo code starts with the declaration of the global variables that are used through the whole program. In the model, four global variables are declared: `x0`, `x1`, `alpha` and `n-circles` (there usages are explained later). After the global variables are declared, the agents must be created. To make these agents, the command `breeds` is used. For this simulation the agents are called circles, because in a first approximation a crater can be modeled as a circle. In NetLogo you can set the properties of your agents. To make this possible, you use the command `own`. This model uses `circles-own` to declare the properties associated with the circles, and three variables are declared: `circles-diameter` (set the diameter of the meteor); `circle-ID` (give an identification to each meteor) and `circle-area` (set the area of the meteor). Also the properties for the `patches` can be defined. `Patches` are the environment where these agents move and live. In the model the command `patches-own` set these properties, and declare the variable `patch-ID`, that will mark were a circle impact the surface, in other words, this will make the craters.

```
globals [x0 x1 alpha n-circles]

breeds [circles]

circles-own [circle-diameter circle-ID circle-area]

patches-own [patch-ID]
```

Then the functions of the model are defined. The following function, named `setup`, clears the environment, to make sure that the surface is clean. Then it sets the range of the crater sizes from `x0` to `x1`. After this, the model creates the meteors with random sizes at random positions, with the variable `n-objects`. The variable `alpha` determines the power-law distribution of the craters which implies that the probability of big impacts is small and the probability of small impacts is large.

```
to setup
 ca
 no-display
 set x0         1.0       ; min. diameter (arbitrary units)
 set x1         35.0      ; max. diameter (arbitrary units)
 set alpha      4.3       ; power-law slope
 set n-circles  100000    ; quantity of impacts
 set-default-shape circles "circle"

 create-custom-circles  n-circles
 [
```

```
  setxy (random (2 * screen-edge-x)) (random (2 * screen-edge-y))
    set circle-diameter power-law2
    set circle-ID who
    set size circle-diameter
 ]

 if file-exists? "powerlaw.dat"
 [
   file-delete "powerlaw.dat"
 ]

 file-open "powerlaw.dat"

 ask circles
 [
  file-print circle-diameter
 ]

end
```

The next function, `to go`, analyzes the surface after the impacts. A crater is completely erased if is covered completely by a bigger impact. The impact process is repeated a certain number of times and the surviving craters are counted as a function of the size. The diameters of the surviving craters are saved on a file to make a log-log plot which is compared with the observed data.

```
to go

   ask circles
   [
      ask circles in-radius (circle-diameter / 2.0)
      [
         if (who <
             value-from myself [who]) and
            (((circle-diameter / 2.0) + (distance myself))<
                 value-from myself [circle-diameter / 2.0])
                and
                (circle-diameter < 5.0)
         [
           die
         ]
      ]
   ]

   type "n-circles (after):" print (count circles)

  if file-exists? "obliteration.dat"
  [
    file-delete "obliteration.dat"
  ]

  file-open "obliteration.dat"

  ask circles
  [
     file-print circle-diameter
  ]

  file-close-all
  print "**** SAVE FINISHED ****"
end
```

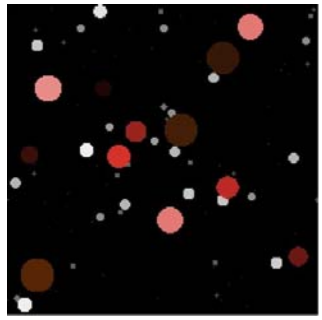

Figure 2. Surface with craters in the computational model

4. Results

Figure 2 shows a typical image obtained from the simulation. In this image, each circle represents a crater of a certain diameter. Once the craters are generated the algorithm takes account of the obliteration process resulting in a reduction in the number of craters. From the resulting (obliterated) image a CSFD plot is constructed which is shown in Figure 3; where the dashed line corresponds to the power-law distribution and the continuous line correspond to the effect of obliteration. From the simulated curve is clear that the break in the slope can be explained by the obliteration process.

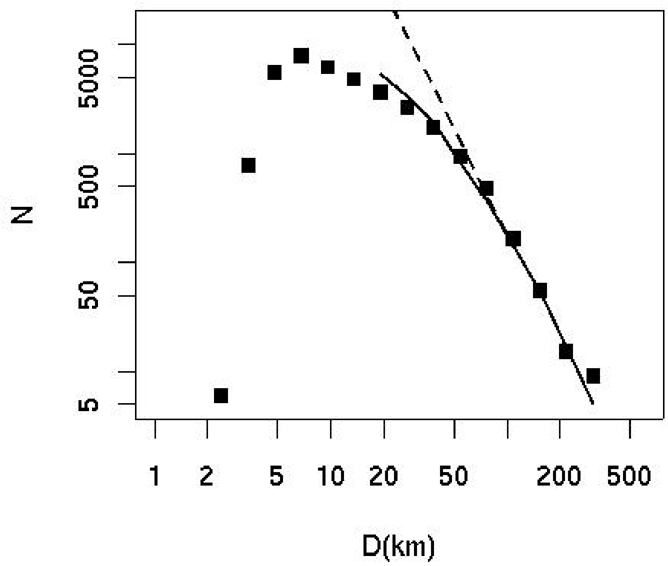

Figure 3. Results from a simulation with 100,000 impacts

5. Conclusion

A computational model was developed to simulate impacts (e.g. meteors, comets) in the Martian surface and reproduce the double slope observed in the crater-size frequency distribution. It was constructed by the generation of impacts with randomly (power-law) distributed diameters falling at uniformly random positions. The algorithm considered situations where the falling impacts erased the craters already in the surface. The computational model shows that the process of erasing due to other impacts (i.e. obliteration) is an important process and it’s able to explain the double slope observed in the range from 20 to 50 Km.

6. Acknowledgements

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