MINIMIZATION IN INCOMPRESSIBLE FINITE ELASTICITY:– NECESSARY CONDITIONS AT THE BOUNDARY

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The boundary-value problems of finite elastostatics are often formulated as energy-minimization problems, and thus as multiple-integral problems in the Calculus of Variations. The condition of *quasiconvexity* for such problems was first introduced by Morrey. Quasiconvexity is equivalent to the lower semicontinuity, in a certain topology, of the integral functional. Quasiconvexity at interior points is a necessary condition for a strong relative minimizer.

For compressible finite elasticity, the condition of *quasiconvexity at the boundary* at boundary points at which traction data is prescribed, was introduced by Ball and Marsden . It, too, is a necessary condition for a strong relative minimizer. Subsequently, Simpson and Spector took up the question of the positivity and non-negativity of the second variation quadratic form for such problems. Specifically, they showed that the appropriate version of Agmon's condition, together with the Legendre-Hadamard condition, and a supplementary condition for cases in which the Legendre-Hadamard quadratic form has zeros, comprise a set of conditions necessary and sufficient for the non-negativity of the second variation. In a recent paper, Mielke and Sprenger have given an elegant, purely algebraic version of Agmon's condition.

Here, I consider the case of *incompressible* elasticity, and develop analogs to the results of Ball and Marsden, and to those of Simpson and Spector in this setting.

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