Symmetry and Degree Theory

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Abstract

There has been recent interest in the interplay of degree theoretic arguments and nonlinear elasticity theory. In particular the degree is useful in studying the existence and branching properties of solutions of the boundary value problems of elasticity. In this talk we shall examine the effects of symmetry on degree theory. Assuming certain equivariance properties for the stress tensor and any imposed body forces, surface displacements or tractions as well as symmetries in the physical shape of the body, we look into how these affect the application of the degree theory. In particular we examine the model situation of a rectangular elastic slab in three dimensions with imposed surface tractions on top and lubricated walls constraining the sides. This imposes a symmetry group on the material and its properties and on the corresponding boundary value problem. The resulting equivariance reduces the problem to a reduced subspace where symmetric solutions reside. We also show how symmetry can be used effectively to deduce the various Fredholm and spectral properties of the linearized equations that are necessary for application of the degree.