Periodic Water Waves with Vorticity

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Abstract

Consider classical inviscid water waves with non-trivial vorticity, with a free surface under the influence of gravity over a flat bottom. Then there exist global continua of 2D periodic traveling waves. The governing equations are Euler's and the pressure is constant on the free surface. There is a global continuum of such solutions for every choice of speed c, spatial period L, relative mass flux p_0 and vorticity function γ that satisfy a certain explicit inequality. The inequality is satisfied for any c, L and γ provided p_0 is sufficiently small. The waves are symmetric around each crest, the period is the distance between successive crests, and the profiles are monotone between crest and trough. Each continuum extends from flat waves all the way to waves that have stagnation points. Thus there exist many rotational periodic traveling waves of large amplitude. These results have been obtained as joint work with Adrian Constantin. Almost all the previous work on this problem, notably by Stokes, Nekrasov, Levi-Civita, Keady, Norbury, Amick and Toland, has been restricted to irrotational flows.